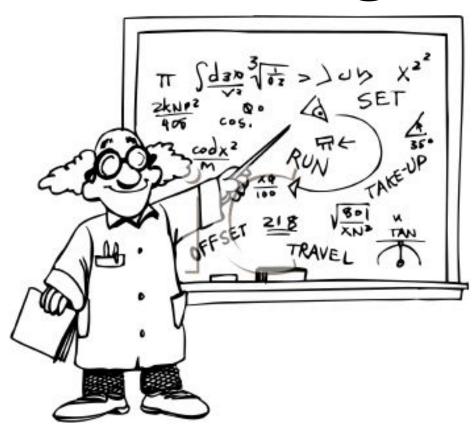
Multiplication Fact Strategies



Wichita Public Schools

Curriculum and Instructional Design Mathematics

> Revised 2014 KCCRS version

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Research Connections

Where Do Fact Strategies Fit In? Adapted from Randall Charles

Fact strategies are considered a crucial second phase in a three-phase program for teaching students basic math facts.

The first phase is **concept learning**. Here, the goal is for students to understand the meanings of multiplication and division. In this phase, students focus on actions (i.e. "groups of", "equal parts", "building arrays") that relate to multiplication and division concepts.

An important instructional bridge that is often neglected between concept learning and memorization is the second phase, <u>fact strategies</u>. There are two goals in this phase. First, students need to <u>recognize there are clusters</u> of multiplication and division facts that relate in certain ways. Second, students need to <u>understand those relationships</u>. These lessons are designed to assist with the second phase of this process. If you have students that are not ready, you will need to address the first phase of concept learning.

The third phase is <u>memorization</u> of the basic facts. Here the goal is for students to master products and quotients so they can recall them efficiently and accurately, and retain them over time.

Learning86, January

Teaching Student-Centered Mathematics John Van de Walle, Jennifer Bay-Williams, LouAnn Lovin, Karen Karp

When students count on their fingers or make marks in the margins they have not mastered their facts because they have not developed efficient methods of producing a fact answer based on number relationships and reasoning. Drilling inefficient methods does not produce mastery!

Over many years, research supports the notion that basic fact mastery is dependent on the development of reasoning strategies. These reasoning strategies are essential to fact development.

Guided invention is an effective research-informed method for fact mastery. Teachers should design sequenced tasks and problems that will promote students' invention of effective strategies. Then, students need to clearly articulate these strategies and share them with peers. This sharing is often best carried out in think-alouds, in which student talk through the decisions they made and share counterexamples.

Effective Drill and Practice

- 1. Avoid inefficient practice. Practice will strengthen strategies and make them increasingly automatic. Do not subject any student to fact drills unless the student has developed efficient strategies for the facts being practiced.
- 2. Individualize practice. Different students will bring different number tools to the task and will develop strategies at different rates. This means there are few drills that are likely to be efficient for a full class at any given time. That is why we need to create a large number of practice activities promoting different strategies and addressing different collections of facts.
- 3. Practice strategy retrieval. When students are involved in a drill exercise that is designed to practice a particular strategy, it is likely they will use that strategy. Organize the students' practice problems according to a selected strategy.

Teaching Student-Centered Mathematics: Volume 2, Van de Walle, p. 94 – 95

Three Steps on the Road to Fluency with Basic Facts

Kim Sutton

- Teach for Understanding
 - Multiplication
 - Repeated addition
 - Area
 - Division
 - Repeated subtraction
 - Area to length of sides
- ❖ Teach in a meaningful sequence, then practice!

Limitations and Risks of Timed Mathematics Tests

Jennifer Bay-Williams & Gina Kling

Timed tests offer little insight about how flexible students are in their use of strategies or even which strategies a student selects. And evidence suggests that efficiency and accuracy may actually be negatively influenced by timed testing. A study of nearly 300 first graders found a negative correlation between timed testing and fact retrieval and number sense (Henry and Brown 2008). Children who were frequently exposed to timed testing demonstrated lower progress toward knowing facts from memory than their counterparts who had not experienced as many timed tests. In fact, growing evidence suggests that timed testing has a negative impact on students (Boaler 2012, Henry and Brown 2008, Ramirez et al. 2013).

(from *Teaching Children Mathematics*, April 2014, pp 488 – 497)

KCCRS Required Fluencies

	K –	2 grade ba	3 – 5 grade band				
		d Subtraction (em solving)	concepts,	 Conceptual Understandings that lead to Fluency: Multiplication and Division of Whole Numbers and Fractions (concepts, skills, problem solving) 			
Required Computational/ Procedural Fluency	Kindergarten K.OA.5 – add/subtract within 5	1 st grade 1.OA.6 – add/subtract within 10	2 nd grade 2.OA.2 – add/subtract within 20 2.NBT.5 – add/subtract within 100	within 100 3.NBT.2 –	4 th grade 4.NBT.4 – add/subtract within 1,000,000	5th grade 5.NBT.5 – multi-digit multiplication	

Defining Fluency

Jennifer Bay-Williams & Gina Kling (from <u>Teaching Children Mathematics</u>, April 2014) A variety of interpretations exist for what procedural fluency (in general) and basic fact fluency (specifically) mean. Fortunately, recent standards, research, and reports provide a unified vision of what these terms signify. The Common Core State Standards for Mathematics (CCSSM) document describes procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (CCSSI 2010, p. 6). Likewise, Baroody (2006) describes basic fact fluency as "the efficient, appropriate, and flexible application of single-digit calculation skills and . . . an essential aspect of mathematical proficiency" (p. 22). These definitions reflect what has been described for years in research and standards documents (e.g., NCTM 2000, 2006; NRC 2001) as well as CCSSM grade-level expectations related to basic facts (see table 1).

Notice that the CCSSM expectations use two key phrases; the first is to fluently add and subtract (or multiply and divide), and the second is to know from memory all sums (products) of two one-digit numbers. To assess basic fact fluency, all four tenets of fluency (flexibility, appropriate strategy use, efficiency, and accuracy) must be addressed. Additionally, assessments must provide data on which facts students know from memory. Timed tests are commonly used to supply such data—but how effective are they in doing so?

Fluency: Simply Fast and Accurate? I Think Not!

Linda Gojak (NCTM Past-President) – from *NCTM Summing It Up*, Nov. 1, 2012 Our students enter school with the misconception that the goal in math is to do it fast and get it right. Do we promote that thinking in our teaching without realizing it? Do we praise students who get the right answer quickly? Do we become impatient with students who need a little more time to think? As we strive for a balance between conceptual understanding and procedural skill with mathematical practices, we must remember that there is a very strong link between the two. Our planning, our instruction, and our assessments must build on and value that connection. Fluency entails so much more than being fast and accurate!

Overview

Fact Strategies

Multiplication	Description of the Strategy	Van de Walle
Doubles	Facts that have 2 as a factor are equivalent to the addition doubles. (Example: 2 x 7 is double 7)	Vol. 2 pg. 88 new: Vol 2 pg 138
Fives	Facts with 5 as a factor.	Vol. 2 pg. 88-89 new: Vol 2 pg 138
Zeroes and Ones	Thirty-six facts have at least one factor that is either 0 or 1.	Vol. 2 pg. 89 new: Vol 2 pg 138
Tens	Facts with 10 as a factor	Vol. 2 pg. 116
Nines	The tens digit of the product is always one less than the "other" factor (the one other than 9), and the sum of the two digits in the product is always 9. So these two ideas can be used together to get any nine fact quickly.	Vol. 2 pg. 89-90 new: Vol 2 pg 139
Square Numbers	Facts where both factors are the same digit. A concrete representation can be made with color tiles. The shape that appears is a square – this is why they are called square numbers. (Example: $3 \times 3 = 9 \text{ OR } 3^2 = 9$, so when color tiles are placed in a 3 by 3 configuration a square is formed.)	
Helping Facts Double and double again	Applicable to all facts with <u>4</u> as one of the factors. (Example: 4 x 6. Double 6 is 12. Double again is 24.)	Vol. 2 pg. 91 new: Vol 2 pg 140
Helping Facts Double and one more set	Applicable to all facts with <u>3</u> as one of the factors. (Example: 3 x 7. Double 7 is 14. One more set of 7 is 21.)	Vol. 2 pg. 91 new: Vol 2 pg 140
Helping Facts Half then double	If either factor is even, a half then double strategy can be used. Select the even factor and cut it in half. If the smaller fact is known, that product is doubled to get the new fact. (Example: 6 x 8. {half} 3 x 8 is 24. Double 24 is 48.)	Vol. 2 pg. 91 new: Vol 2 pg 140
Helping Facts Add one more set	Can be used with any fact. (Example: 6 x 7; 5 sevens are 35. One more set of 7 is 42.)	Vol. 2 pg. 91-92\ new: Vol 2 pg 140

Making Meaning for Operations (Teacher Use Only) Structures for Multiplication and Division Problems ~VDW 56-60 vol. 2

This section is provided for teachers so they better understand how to help students develop operational sense to connect different meanings of multiplication and division to each other. This will enable students to effectively use these operations in real-world settings. These problem structures are not intended for students but will help you as the teacher in formulating and assigning multiplication and division tasks that cover all situation types.

Math Activity: Modeling Word Problems

- 1. Model each of the following five problems with cubes or other counters. After you have acted out the problems with a concrete model, write an arithmetic sentence for each one.
 - a. This month Mark saved 5 times as much money as last month. Last month he saved \$7. How much money did Mark save this month?
 - b. If apples cost 7 cents each, how much did Jill have to pay for 5 apples?
 - c. This month Mark saved 5 times as much money as he did last month. If he saved \$35 this month, how much did he save last month?
 - d. Mark bought some ice cream and toppings for a party. He bought 5 different types of ice cream and 7 different types of toppings. How many different ice cream combinations can Mark make?
 - e. Jill bought apples at 7 cents apiece. The total cost of her apples was 35 cents. How many apples did Jill buy?
- 2. How are these five problems alike? How are they different?
- 3. What connections do you see between the five problems and the information presented on the chart, *Common Multiplication and Division Situations* (pg. 89 in CCSS)?

Making Meaning for Operations

Common Multiplication and Division Situations (pg 89 in CCSS)

Grade level identification of introduction of problems taken from OA progression

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	$3 \times ? = 18; 18 \div 3 = ?$	$? x 6 = 18; 18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays ⁴ , Area ⁵	There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a × b = ?	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Multiplicative compare problems appear first in Grade 4 (green), with whole number values and with the "times as much" language from the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit language change the subject of the comparing sentence ("A red hat costs n times as much as the blue hat" results in the same comparison as "A blue hat is 1/n times as much as the red hat" but has a different subject.)

Multiplication Fact Strategies Assessment Directions and Analysis

As students are taking the assessment you will need to observe your students to see if they are using counting strategies (fingers, marks, verbal counting, etc.) instead of reasoning strategies. This assessment gives the teacher one "view" of what a student can do with fact strategies. This section is broken down so that each box focuses on fact strategies that are appropriate for the facts given in the box.

Directions for the Assessment

- 1. Fill out the Fact Strategies View Recording Sheet (p. 13) so that all of your students are listed. (A clipboard or something similar could be used so that you are able to make the appropriate marks quickly.)
- 2. Explain to the students they will be taking a quick assessment that will be used to assist you in identifying their fact strategy needs.
- 3. Hand out the one page assessment of fact strategies (p. 12) to each student.
- 4. Explain that they will be working in <u>only one box at a time</u> and that you will be making notes as they complete the box. Give students about 7 to 8 seconds to answer the facts, but do not say you are timing them. Let students know that if they finish the box before you have asked everyone to stop, you would like for them to place their pencil down on their desk. When you say stop, have them put their pencils up in the air.
- 5. Tell students to look at Box #1. When you are certain that all students are focused on the correct box, ask them to begin. Make a check mark next to students' names on your recording sheet if you see they are using their fingers, tally marks, verbally counting, etc. Ask the students to stop (put pencils in the air) when time is up.
- 6. Ask students to look at Box #2. Have them begin and continue to do the same markings.
- 7. Use the same procedure for all eight boxes.
- 8. When you are finished with the 8th box, ask the students to fill in the bottom of the paper. You may read the directions to them and allow them time to fill it out or they may fill it out on their own.

Analysis of the Assessment

- 1. The **Fact Strategies View** page is broken into sections so that the different boxes focus on strategies that are appropriate for those particular number sentences. Strategies that can be used for the examples in the box:
 - Box #1 Doubles
 - Box #2 Five Facts
 - Box #3 Zeroes and Ones
 - Box #4 Nines
 - Box # 5 Square Numbers
 - Box #6 Double and Double Again
 - Box #7 Double and One More Set
 - Box #8 Half then Double
- Collect the students' papers. Look at each box and check to see if students have missed 2 or more problems in that box. If they have, then they need to work on the strategies that are listed for that particular box. Make a note on the Fact Strategies View recording page.
- 3. Read what students wrote as the easiest and the hardest problems to do. This will give you an idea if the students are consciously using strategies or not, and if they are, which strategies they are most comfortable in using. Make note of this on your recording sheet.

And once I had a teacher who understood. He brought with him the beauty of mathematics. He made me create it for myself. He gave me nothing, and it was more than any other teacher has ever dared to give me.

-Cochran (1991)



Fact Strategy View

$$2 \times 4 =$$

$$2 \times 7 =$$

$$5 \times 2 =$$

$$8 \times 2 =$$

$$5 \times 6 =$$

$$8 \times 5 =$$

$$5 \times 7 =$$

$$5 \times 4 =$$

$$3 \times 5 =$$

$$0 \times 6 =$$

$$7 \times 1 =$$

$$1 \times 5 =$$

$$0 \times 8 =$$

$$1 \times 3 =$$

$$7 \times 9 =$$

$$9 \times 6 =$$

$$3 \times 9 =$$

$$4 \times 4 =$$

$$7 \times 7 =$$

$$8 \times 8 =$$

$$5 \times 5 =$$

$$8 \times 4 =$$

$$4 \times 7 =$$

$$4 \times 3 =$$

$$6 \times 4 =$$

$$4 \times 8 =$$

#7

$$8 \times 3 =$$

$$3 \times 7 =$$

$$3 \times 5 =$$

$$3 \times 8 =$$

#8

$$8 \times 6 =$$

$$6 \times 7 =$$

$$7 \times 8 =$$

$$6 \times 8 =$$

$$7 \times 6 =$$

Circle the box that was the easiest for you to complete.

Box #1

Box #2

Box #3

Box #4

Box #5

Box #6

Box #7

Box #8

Why was it the easiest?

Circle the box that was the hardest for you to complete.

Box #1

Box #2

Box #3

Box #4

Box #5

Box #6 Box #7

Box #8

Why was it the hardest?

Multiplication Fact Strategies Recording Sheet

Put a ✓ in the box if the student has trouble with the problems in the box. (Ex: made tally marks, used fingers, or counted verbally to solve the problems, OR missed 2 or more problems in that box.)

Name	Box 1	Box 2	Box 3	Box	Box 5	Box 6	Box 8	Comments

Student Discourse

Students must cognitively engage with the strategies by discussing them with their peers and their teacher. Talk is essential for full understanding about numbers and how the operations work. Encourage students to verbalize their reasoning. Encourage them to add on to another's thinking or to agree or disagree with each other. All of this must be done in a safe and supportive environment that values everyone's contributions. Two resources that we recommend to support these expectations are <u>Classroom Discussion in Math</u> by Suzanne Chapin and <u>Number Talks</u> by Sherry Parrish. The first resource provides the background for all student discourse in a mathematics classroom and comes with a DVD that shows discourse in action. The second resource explains how this can be done with a focus on mental computation and strategies. This resource also has a DVD for teachers to use.

Student discourse is a part of the expectations for the Standards for Mathematical Practice in the KCCRS.

- 1. Make sense of problems and persevere in solving them. Strategies help students see that math does make sense. There is an order and a process, but they need to talk it out with others in order to enhance and deepen their understanding.
- 2. Reason abstractly and quantitatively. Talking with peers and teachers leads thinking from the concrete to the abstract to increase their flexibility in reasoning mathematically.
- 3. Construct viable arguments and critique the reasoning of others. The heart of student discourse.
- 4. *Model with mathematics*. During discussion students will need to also show their thinking by writing the equations and to interpret abstract mathematics into context.
- 5. Use appropriate tools strategically. Strategies are tools for mental computation. Exploring the effectiveness of these strategies helps determine when each strategy is most appropriately used and when it is not.
- 6. Attend to precision. This standard is not only about being precise with mathematical symbols but also with math vocabulary which is essential in making thinking understood when discussing with peers.
- 7. Look for and make use of structure. Once the structure of specific strategies are understood and clarified then students are able to effectively apply appropriate strategies.
- 8. Look for and express regularity in repeated reasoning. In order to fully understand each strategy students need to see it repeatedly, and in different ways, in order to make the generalizations that are necessary for future application of that strategy.

Question Stems:

- How does this strategy work?
- What kinds of problems does this strategy help solve?
- When is this strategy not appropriate?
- How does this strategy compare with a previous strategy we explored?
- Is there more than one strategy that can be used with this problem?

The Array

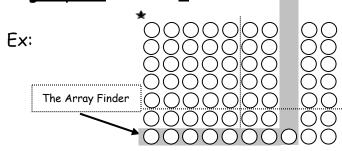
Finder

Using the Array Board and the Array Finder As your students become more confident in using concrete objects to show arrays, you will want to move them to the pictorial stage. These pages will help your students in making the connection from the concrete to the abstract. Make sure each student has an Array Finder (the pattern

for the Finder is the border of this page) and an Array Board that is a sheet of 10 by 10 dots.

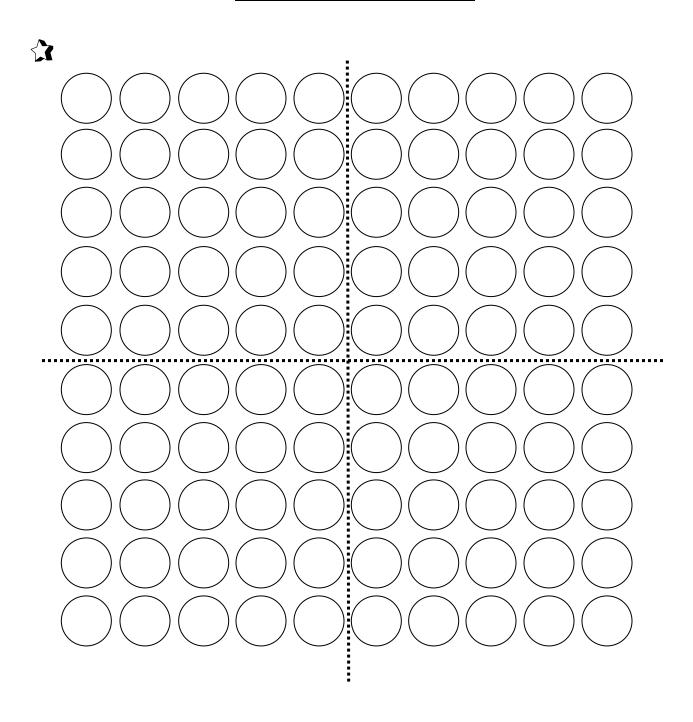
Model for your students how to use the Finder. Have them take their "L" shaped Finder and rotate it so that it looks like this: Inside corner

Put the inside corner of the Finder at the top left-hand side of the array where you will see a star. You should only see the "Starting Star" in your Finder; no other objects should be seen in the Finder. If your problem is " 6×7 ", move the Finder down 6 rows (groups) and over 7 columns. Have the students count the number of dots in the Array Finder's viewing area, and they will have the array for "6 groups of 7" or "6 x 7"!



The lines in the array make counting the dots easier and often suggest the use of the easier fives facts as helpers. For example, 6 x 7 would be 5 x 7 plus one more group of 7 (35 + 7 = 42).

Array Board



Teaching Student-Centered Mathematics Grades K-3 by John A. Van de Walle

Multiplication Table

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Every student should have a multiplication table. Each time they master a set of facts, they should color in the multiplication facts they now know (or write them into the blank one). This allows them to celebrate the facts they know and view the facts they have yet to learn.

The sets of mini-multiplication tables on the next page can be used to see the pattern of each strategy.

Mini-Multiplication Tables

X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

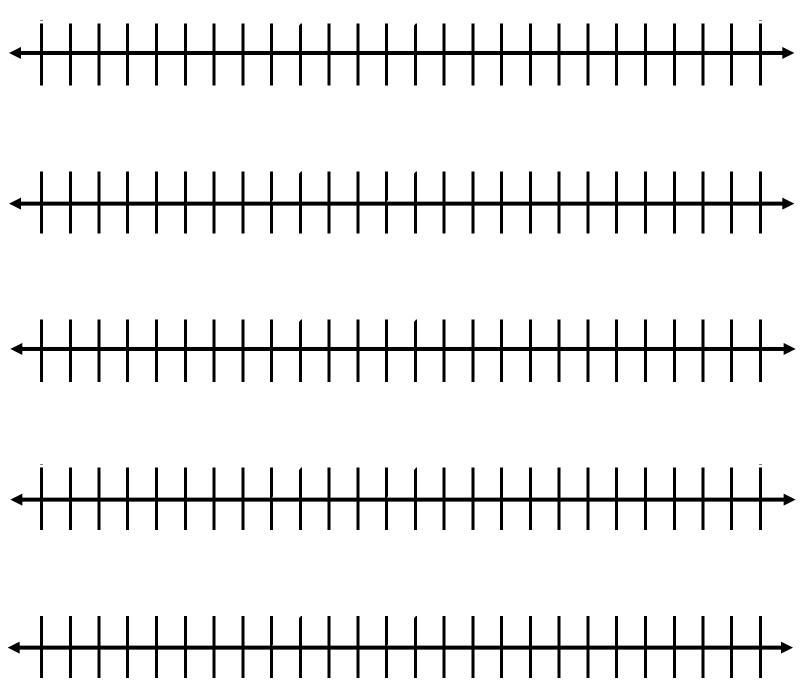
X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Build Your Multiplication Table

X	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

Number Lines



Number lines are another tool students should have the opportunity to use when demonstrating their understanding of computation. Multiplication would be demonstrated with sets consisting of the same number (as in repeated addition). For example: 4 x 2 means four groups of 2. That could look like:

2 2 2 2 2

0 1 2 3 4 5 6 7 8 9 10
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Fact Strategy Punch Cards

Multiplication Fact Strategies Punch Card					
Student:					
Doubles					
Fives					
Zeroes and Ones					
Tens					
Nines					
Squared Numbers					
Double and Double Again					
Double and One More Set					
Half Then Double					

Multiplication Fact Strategies Punch Card						
Student:	Student:					
Doubles						
Fives						
Zeroes and Ones						
Tens						
Nines						
Squared Numbers						
Double and Double Again						
Double and One More Set						
Half Then Double						

Every student could have their own punch card. Each time they master a strategy, they make a punch for the strategy that they now know. This lets them celebrate what they've learned and to look ahead to the strategies they have yet to learn. **Note**: Do not display these cards publically. They should be in a folder or other location for just the student to access.

You might want to copy these on Bristol paper for durability.

Miles of Multiples

Cut each strip apart. Continue the pattern by listing the multiples of each number. The point of ellipse is at the point to show that the multiples continue infinitely.

2	4	6	8	10	12	14	16	18	20	•••
3	6	9	12	15	18	21	24	27	30	•••
4	8	12	16	20	24	28	32	36	40	•••
5	10	15	20	25	30	35	40	45	50	•••
6	12	18	24	30	36	42	48	54	60	•••
7	14	21	28	35	42	49	56	63	70	•••
8	16	24	32	40	48	56	64	72	80	•••
9	18	27	36	45	54	63	72	81	90	•••
10	20	30	40	50	60	70	80	90	100	•••
										•••

Doubles Fact Strategy

Students will be working on the facts that have 2 as a factor. These are equivalent to the addition doubles. (Example: 2×7 is double 7, 7 + 7 = 14)

Materials Needed:

- Multiplication chart and student copies
- Colored pencils/crayons
- "Doubling Up" game board (each)
- Deck of Cards (per pair)
- 5 sheets of blank paper (per student)
- Partner Practice page (Bristol—copied back to back) (pairs)

Teaching Fact Activity

Materials: 5 sheets of white copy paper for each student, scissors

Directions:

Let's take a look at a problem and talk about how we might solve it.

Dad said he would double the money earned if I finished mowing by 3:00. I normally earn \$4. How much will Dad pay if I'm done at 2:30? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Lenn has a package of 8 hot dog buns. He picked up another package at the store. How many buns does he now have? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

When working with the facts that have $\mathbf{2}$ as one of the factors, you can use the **Doubles Strategy**. In this strategy, you double the factor that is not the 2. Take one full sheet of paper, cut it in half vertically so you end up with 2 half sheets that are $4 \frac{1}{4} \times 11$.

Now take the half sheets and fold them in half. Open up one of the papers and draw three dots on each side of the paper.



This would make one set of three. Both sides will need the same number of dots. Once you draw three dots on the left side, then you will draw three dots on the right side. This is like addition doubles. Two groups of three is six. Two times three equals six.

You will take 4 more sheets of paper to cut in half the same way. Make all of your "x 2" facts from 1 to 10 on these strips of paper. We just did the threes

together. You may wish to use these when playing some of the games and activities. Ask if there are any questions.

Practice Fact Activity (Approximate time: 15 minutes):

Doubling Up!

Materials: game board and a deck of cards

We are going to learn a game that will be played to help you practice the doubles strategy. It's called **Doubling Up!** You will need a game board for each player and a deck of cards for every pair of students. Take out all of the face cards except for the Kings (these will represent zero). Shuffle the cards and place them face up between the players. Each player will take turns drawing a card. When a card is drawn the player will multiply the numeral on the playing card by 2 and then cover the product on their game board with their card. If the product is already covered then the player will have to place the card in a discard pile and lose that turn. The first player to cover all of the products on their board wins!

Practice Fact Activity

Partner Practice Page

Materials: Doubles practice partner page (Bristol)

Use this page to practice the Doubles. Two people use this page. The person who is going to be the "coach" will hold the paper up between them so that the facts without the answers are facing the person who is the "student". The "student" should read the problem and then give the answer. The "coach" gives feedback as to whether the answer is right or wrong.

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Partner Practice Page

Doubles Strategy

$$4 \times 2 = 2 \times 7 = 2 \times 2 = 2 \times 1 = 6 \times 2 =$$

$$9 \times 2 = 2 \times 3 = 4 \times 2 = 2 \times 5 = 8 \times 2 =$$

$$9 \times 2 = 2 \times 3 = 4 \times 2 = 2 \times 5 = 8 \times 2 =$$

$$4 \times 2 = 2 \times 7 = 2 \times 2 = 2 \times 1 = 6 \times 2 =$$

Partner Practice Page

Doubles Strategy

Answer Key

4 x 2 = 8	2 x 7 = 14	2 x 2 = 4	2 x 1 = 2	6 x 2 = 12
9 x 2 = 18	2 x 3 = 6	4 x 2 = 8	2 x 5 = 10	8 x 2 = 16

2	8	2	6	2	7	1
<u>x6</u>	<u>x2</u>	<u>x5</u>	<u>x2</u>	<u>x3</u>	<u>x2</u>	<u>x2</u>
12	16	10	12	6	14	2
9	2	3	2	2	4	5
<u>x2</u>	<u>×7</u>	<u>x2</u>	<u>x1</u>	<u>×8</u>	<u>x2</u>	<u>x2</u>
18	14	6	2	16	8	10
2	2	8	2	7	2	2
<u>x4</u>	<u>x5</u>	<u>x2</u>	<u>×9</u>	<u>x2</u>	<u>×6</u>	<u>x2</u>
8	10	16	18	14	12	4

9 x 2 = 18	2 x 3 = 6	4 x 2 = 8	2 x 5 = 10	8 x 2 = 16
4 x 2 = 8	2 x 7 = 14	2 x 2 = 4	2 x 1 = 2	6 x 2 = 12

Fives Fact Strategy

Students will be working on the facts that have <u>5 as a factor</u>. Products of facts with five as a factor will end in either a 5 or 0.

Materials Needed:

- Multiplication chart and student copies (use copy from Doubles lesson)
- "Flying High" and student copies
- 10-sided die/deck of cards (each person)
- transparent chip (each person)
- paper clip (pair)

Teaching Fact Activity

Focus: Get out your Multiplication Table and a colored pencil/crayon. Lightly color in the doubles facts once you have them learned. As the days go by, we will see the number of facts to memorize decrease (the spaces not yet shaded).

Directions:

Let's take a look at a problem and talk about how we might solve it.

The class made a mural of everyone's handprints. Jill counted the number of fingers. How many fingers will she have counted on six hands? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

She has counted 30 fingers. How many hands has she counted?

Clock Spinner

Materials: Objects for counting by 5s such as hands, or hundred board and counters, paper and paperclip for each student

Directions:

Provide opportunities for students to practice counting by 5s using a variety of objects and then from memory. Then have students draw a face clock on a sheet of paper (or provide a copy). Using the paper clip spinner, students generate multiplication problems by spinning the spinner. Whatever number the spinner lands on, multiply that number by 5. Repeat.

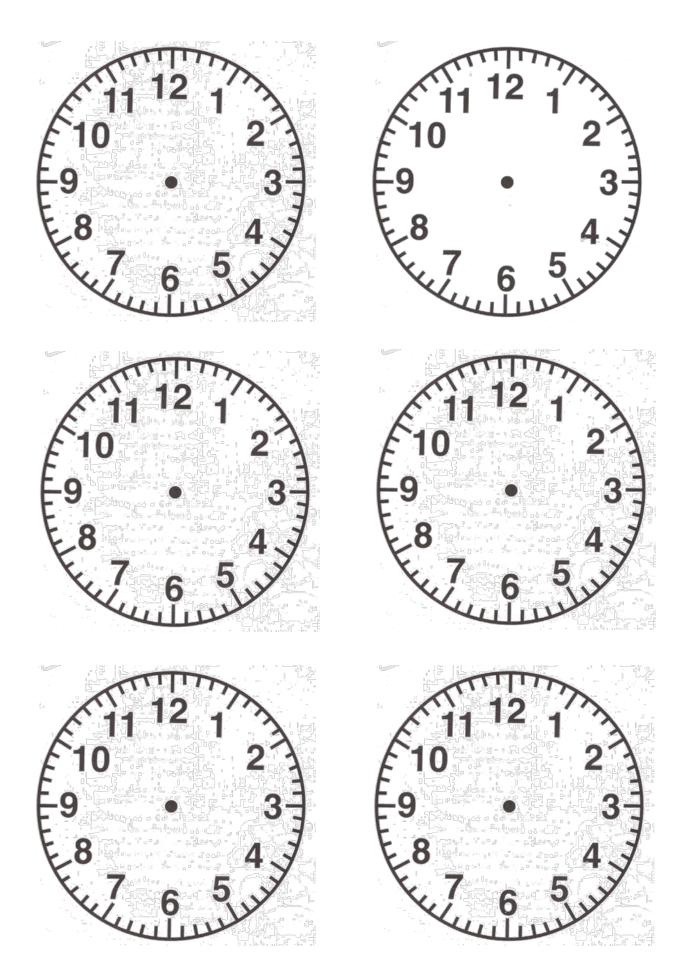
 $9 \times 5 = 45$

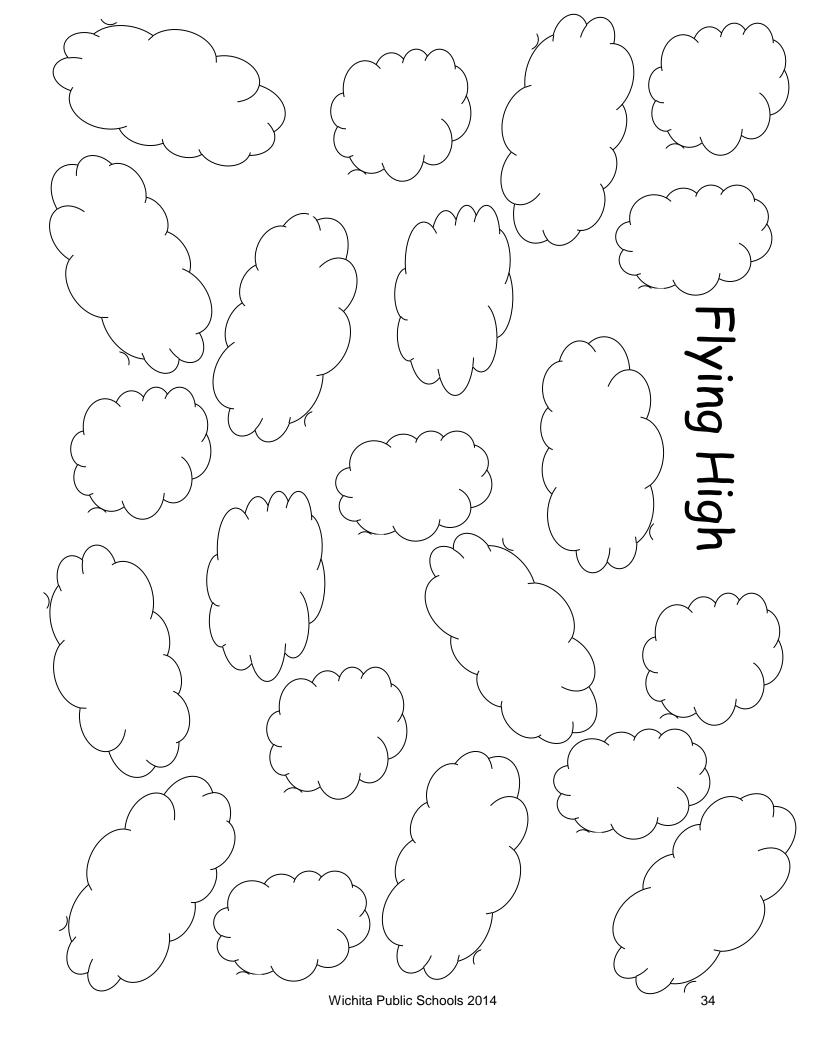
Practice Fact Activity

Flying High

Materials Needed: Flying High, a deck of cards or the 10-sided die On the back of the **Flying High** sheet multiply each number (0-9) by 5 and circle the products (i.e. $0 \times 5 = 0$; $1 \times 5 = 5$) $2 \times 5 = 19$; $3 \times 5 = 15$. . .). Then turn your paper over and write each product (circled number) in each of the clouds. You may have to repeat products until all clouds are filled. Your game board is ready!

(Have the students remove all of the face cards and the 10s, shuffle the deck, and then place it face down in front of them. One student will then take the top card and turn it over to get one of the factors.) Using your deck of cards (or the 10-sided die) to generate a factor, multiply the number on the card by 5. Think what the product will be and then cross that product off (or cover it up with a counter). Only mark one cloud at a time. If all clouds are crossed off for that product, draw a card (or roll the die) to get the next number. Once all clouds are crossed off, call out "I'm Flying High!"





Zeroes & Ones Fact Strategies

Students will be working on the facts that have <u>0 and 1 as a factor</u>.

Materials Needed:

- Multiplication chart and student copies
- "Zeroing in on Being Number One" mats (per pair)
- blank cube (per pair)
- 10-sided die (per pair)
- Partner Practice Page (Bristol—copied back to back)

Focus

Materials: Multiplication tables for each student, colored pencils/crayon, a Multiplication table chart

Mark all of the 5 facts on the multiplication facts once they have been learned.

Directions:

Let's take a look at a problem and talk about how we might solve it.

Milk is sold in 2-cup containers. The store sold out of milk this morning. How many cups of milk are available? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.) What does the number sentence look like?

The cereal company put one toy in each box of cereal. If there are 7 boxes of cereal on the shelf; how many toys would you expect there to be altogether? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Teaching Fact Activity

Directions:

Let's explore what happens when one of the factors is a 0 or a 1. The "x" symbol means "groups of", so the number sentence 1 x 6 means "1 group of 6". When you use the commutative property (or the order property) and switch the numbers around, "6 x 1" means "6 groups of 1". The picture of each one of these is different. (Have students demonstrate with manipulatives both of these facts.)

6 x 1 would look something like: \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc (6 sets with 1 in each set)

1 x 6 would look something like this: (:::) (1 set with 6 in it)

This makes it easy to see why, when you multiply by 1, the other factor is the product. This is called the Identity Property. Put a few "x 1" problems up on the overhead and have the students say the number sentence with the phrase "groups of" for the multiplication sign.

When one of the factors is a 0, such as 0 x 8, then the number sentence is read "0 groups of 8". If you don't have any sets then how many things do you have? (none or zero) If the number sentence is 8 x 0, then it would be read "8 groups of 0". If you have 8 groups of nothing, how many things do you have? (Zero) This is why when you multiply by 0 the answer is always 0. This is called the Zero Property.

Practice Fact Activity (0s and 1s)

"Zero"-ing in on Being Number "One"

Materials: wooden cube, permanent marker, 10-sided die, two counters/chips and game board for each pair of students

Teacher prep: Use a permanent marker and write the number 1 on <u>four</u> sides of the wooden cube and the number 0 on the other two.

Directions:

Each player needs a chip to use as a mover. Place the chips on start and roll to see who goes first (highest roll wins). The first player will roll both the wooden cube and 10-sided die, multiply to say the product, and then move their chip. If a "x 1" problem is rolled, then move the number of spaces of the product. If a "x 0" problem is rolled, then the player is to move to the next nearest "0" on the game board. If a player states an incorrect answer, he/she will stay put. The first person to reach the Winner's Circle is the winner.

Practice Fact Activity

Partner Practice Page

Materials: Fives, Zeros, and Ones practice partner page (Bristol)

Use this page to practice the Fives, Zeros, and Ones. There will be review problems from previous learned strategies. Two people use this page. The person who is going to be the "coach" will hold the paper up between them so that the facts without the answers are facing the person who is the "student". The "student" should read the problem and then give the answer. The "coach" gives feedback as to whether the answer is right or wrong. Demonstrate positive responses to correct answers, such as... "Good job", "Alright", "Way to go", etc.)

"Zero" -ing in on Being Number "One"

Materials: Each pair of students need the following: labeled wooden cube (0 on three sides and 1 on three sides), 10-sided die, and 2 game pieces, and the game board

- 1. Each player must place their game piece on the shoe and roll the 10-sided die to see who goes first (highest roll goes first).
- 2. The first player will roll both the wooden cube and the 10-sided die. That player multiples the numbers rolled and states the equation.
- 3. If correct, that player moves their game piece to the nearest number that matches the product they just named.
- 4. If that player states an incorrect answer, the player does not get to move.
- 5. The second player takes a turn rolling the dice, stating the equation, and moving their game piece along on the board. (More than one player may occupy a space.)
- 6. The player whose product is not found on any space between where he/she is and the end, jumps to the trophy and wins!

"Zero"-ing in on Being Number "One"

Zeroes, Ones and Fives Strategy

$$4 \times 5 = 0 \times 7 = 2 \times 1 = 0 \times 1 = 6 \times 5 =$$

$$9 \times 0 = 5 \times 3 = 4 \times 1 = 5 \times 5 = 8 \times 1 =$$

$$9 \times 5 = 2 \times 3 = 4 \times 0 = 5 \times 5 = 8 \times 5 =$$

$$4 \times 1 = 2 \times 7 = 2 \times 1 = 9 \times 2 = 6 \times 0 =$$

Zeros, Ones and Fives Strategy

Answer Key

4 x 5 = 20	0 x 7 = 0	2 x 1 = 2	0 x 1 = 0	6 x 5 = 30
9 x 0 = 0	5 x 3 = 15	4 × 1 = 4	5 x 5 = 25	8 x 1 = 8

1	8	0	6	5	7	1
<u>×6</u>	<u>×5</u>	<u>x5</u>	<u>x5</u>	<u>×3</u>	<u>×0</u>	<u>×1</u>
6	40	0	30	15	0	1
9	5	3	0	1	4	5
<u>x1</u>	<u>×7</u>	<u>x0</u>	<u>x1</u>	<u>×8</u>	<u>×5</u>	<u>x1</u>
9	35	0	0	8	20	5
0	1	8	5	7	1	5
<u>×4</u>	<u>×5</u>	<u>x1</u>	<u>×9</u>	<u>×5</u>	<u>×6</u>	<u>x2</u>
0	5	8	45	35	6	10

9 x 5 = 45	2 x 3 = 6	4 × 0 = 0	5 x 5 = 25	8 × 5 = 40
4 × 1 = 4	2 x 7 = 14	2 x 1 = 2	9 x 2 = 18	6 x 0 = 0

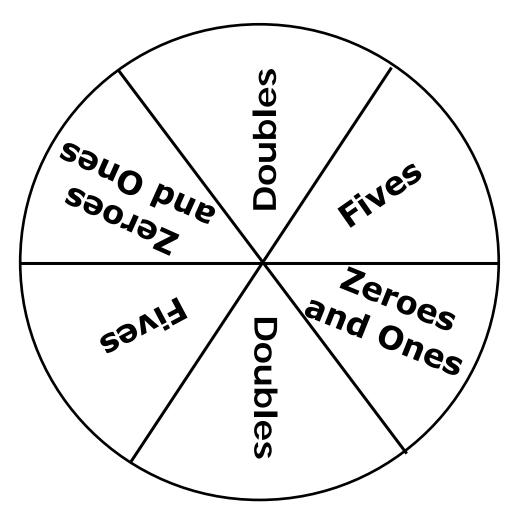
Strategy Focus Review

Students need to practice the strategies that have been taught. They need to start reasoning out which facts are best solved with particular strategies. This should be intentionally practiced and discussed.

Use the spinner below with the facts on the next page. Give each student a copy of the facts page. Place the spinner under the document camera and spin it. The spinner will indicate which strategy the students will focus on for their facts page. If the spinner lands on "Doubles," then the students will identify the facts on their page that can easily be solved using that strategy and circle them.

After a short amount of time, place a copy of the facts page under the document camera and have the students tell you which facts were circled. Have students explain why that fact can be solved with that strategy for the first 3 or 4 given to you.

When facts have been circled and verified that they can be solved using that particular strategy, have the students answer the problems that are circled using the indicated strategy. Have students pair up when finished to check their answers to see if they agree.



Strategy Focus Review (Doubles, Fives, Zeroes and Ones)

4	0	2	5	1
<u>x 5</u>	<u>x 3</u>	<u>x 8</u>	<u>x 7</u>	<u>x 4</u>
7	9	1	9	8
<u>x 2</u>	<u>x 0</u>	<u>x 8</u>	<u>x 2</u>	<u>x 5</u>
3	6	2	5	8
<u>x 5</u>	<u>x 2</u>	<u>x 4</u>	<u>x 5</u>	<u>x 0</u>
1	5	2	5	1
<u>x 7</u>	<u>x 2</u>	<u>x 3</u>	<u>x 0</u>	<u>x 1</u>
4	6	2	2	9
<u>x 0</u>	<u>x 5</u>	<u>x 9</u>	<u>x 2</u>	<u>x 5</u>

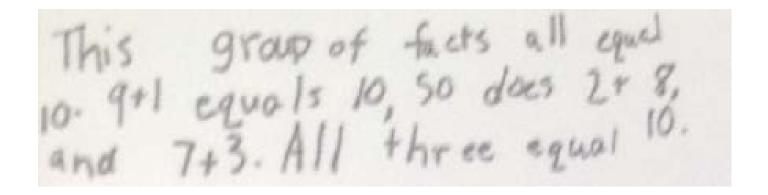
Strategy Focus Review

Students need to be able to discuss and write about their reasoning with strategies. This activity continues the focus on strategies but engages the students cognitively by writing about their reasoning for a particular strategy for a specific fact or set of facts.

Use the previous page (pg. 42) to create a set of cards for each student or pair of students. These cards will be sorted on the strategy mat on the following page. After the cards have been sorted, the students will check their thinking with a partner and then discuss any discrepancies between them. The students need to explain to their partner why they choose that particular strategy that may be different from their partner. This doesn't mean their strategy use was incorrect (because one fact could be solved by more than one strategy) but listening to another's reasoning could let them see another way to attack problems.

Students may switch facts around while they are talking with their partners or they may decide to leave them. The facts are finally written, taped or pasted onto the chart (if students are taping or pasting you will want to enlarge the chart onto 11 x 17 paper to allow for the cards and the writing at the bottom) and then one strategy is chosen to explain why those facts where chosen to be solved with that strategy.

Here is a student example of their reasoning – although this is with an addition strategy, you can see the thinking the student went through in using a strategy to solve the facts.



Strategy Sort

Choose one strategy and explain v	Doubles
Choose one strategy and explain why those facts are solved with the chosen strategy:	Zeroes and Ones
chosen strategy:	Fives

Tens Fact Strategy

Students will be working on the facts that have 10 as one of the factors.

Materials Needed:

- Hundred Chart (each)
- Hundred Chart
- Deck of Cards (pair)

Focus:

Look at the number of facts that have been mastered on the large multiplication chart at the front of the room. Review the **fives**, **zeroes** and **ones** from the previous lessons.

Teaching Fact Strategy

Counting by 10s

Materials: hundred chart for each student

Each student needs a hundred chart. Have each student color in the numbers that they say when counting by 10. Students can practice counting verbally by 10s. A fun way to review this is to include motion while counting by 10s. For example: Stand when you say 10, 30, 50, 70, and 90. Sit when you say 20, 40, 60, 80 and 100.

Directions:

Let's take a look at a problem and talk about how we might solve it.

Peggy has 8 dimes. How much money does she have in cents? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Hot dogs come in packages of 10. We need 60 hot dogs for the class party. How many packages of hot dogs should be bought? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Teaching Fact Strategy

Tens Place Think Aloud

Materials: overhead hundred chart for the teacher

Directions:

Often times a teacher can teach a concept by simply showing and telling the students what they are thinking. This is called a "Think Aloud". Example: use the 100 board to illustrate that **2 groups of 10** is $\underline{\textbf{2}}$ 0, **4 groups of 10** is $\underline{\textbf{4}}$ 0, etc (each row is a set of 10).

Practice Fact Strategy

Draw of 10

Materials: deck of cards for each pair of students (without face cards)

Directions:

Students can work in pairs. Each pair needs 1 deck of cards. Split the deck in half. Each partner gets one half of the deck. Each partner turns over a card. If I turn over a 7, I say 7 groups of 10 equals 70. My partner turns over a 4, and says, 4 groups of 10 equals 40. The person with the largest product wins the round and takes both cards. Play continues until all the cards have been played.

Variation: the person with the smallest product is the winner.

Practice Fact Strategy

Show Me the Product

Materials: hundred chart for each student

Directions:

Students are given an equation in verbal or written form. Students use a pointer to show the correct product on their individual hundred chart. For example, the teacher says 6 x 10. The students point to 60 on their hundred chart. Repeat using different factors with 10.

Variation: Use white boards, show me pockets with digit cards, numeral slides, etc.

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Nines Fact Strategy

Students will be working on facts in which 9 is one of the factors.

Materials Needed:

- Multiplication chart and student copies (use copy from Day 1)
- "Handy Nines"
- 10-sided die (per pair)
- Triangle flash cards (each student) (Bristol)
- Partner Practice Page (per pair) (Bristol—back to back)

Teaching Fact Activity:

Materials: mini-multiplication chart, green pencil/crayon Have each child get his or her copy of the mini-multiplication chart

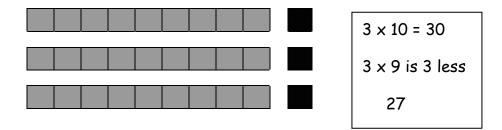
Directions:

Let's take a look at a problem and talk about how we might solve it.

The floor is 9 feet long by 7 feet wide. How many square feet cover the floor? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols. Using color tiles and/or graph paper are practical for building arrays to solve this problem.)

Jessie picked 36 apples. Jessie picked 9 times as many apples as Donna. How many apples did Donna pick? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols. See "Making Meaning of Operations" p.7 to assist with understanding how this problem could be approached.)

A strategy for the nines that is easy to use, is related to the tens strategy. Notice that 3 x 9 is almost like 3 x 10 (just one set less). This can be easily modeled with cubes. Place them in 3 rows of 10 cubes each. The last cube in each row can be a different color and/or a little detached from the rest of the cubes in its row.



(<u>Teaching Student-Centered Mathematics: Volume 2</u>, Van de Walle, p. 90) Next, examine the multiplication chart to find a pattern with multiples of nine. [Give students time to see the patterns --tens place increasing, ones place decreasing; sum of the digits in the product add up to nine (digital root is 9) -- when looking at the nines column on a multiplication table.] Students may also share other tricks to remembering their nine's facts.

For example: the tens-place digit is one less than the factor that is not a nine, so the ones-place digit is the other addend that makes nine

$$(9 \times 7 = 63)$$
One less makes nine

There is also a "trick" with your hands that students may also know. Directions for how to do this with the "Handy Nines" is as follows: Lay both hands flat (palms down) on the table in front of you. Number each finger 1-10 from left to right. If the equation is 5 x 9 = , count to your 5th finger and tuck it under. This should be the thumb on your left hand. How many fingers do you see to the left of the "tucked' finger? [4] How many fingers to the right? [5]. The answer is 45. (They will want to see that it works with other x9 facts, so give a little time to check it out.)

WARNING: Although the nines strategies (tricks) can be quite successful, it also can cause confusion. Because two separate rules are involved and a conceptual basis is not apparent, students may confuse the two rules or attempt to apply the idea to other facts. It is not, however, a "rule without reason". It is an idea based on a very interesting pattern that exists in the base-ten numeration system. One of the values of patterns in mathematics is they help us do seemingly difficult things quite easily. The nifty-nine pattern illustrates clearly one of the values of pattern and regularity in mathematics.

Practice Fact Activity:

Partner Practice Page

Materials: Nines practice partner page (Bristol)

Use this page to practice the Nines. There will be review problems from previous learned strategies included. Two people use this page. The person who is going to be the "coach" will hold the paper up between them so that the facts without the answers are facing the person who is the "student". The "student" should read the problem and then give the answer. The "coach" gives feedback as to whether the answer is right or wrong. Demonstrate positive responses to correct answers, such as...Good Job, Alright, Way to go, etc.)

Practice Fact Activity

Triangle Flash Cards

Materials: triangle flash cards (from the EM resources) Give the students the triangle flash cards.

Directions:

- 1. Find all of the triangles that have a nine as a factor.
- 2. Hold the triangle so that your thumb covers the product (the underlined number).
- 3. Multiply the two factors shown to identify the product that is covered.

Triangle flash cards for other factors will be used later.





Nines Strategy



$$9 \times 2 = 9 \times 7 = 9 \times 3 = 9 \times 1 =$$

$$9 \times 7 =$$

$$9 \times 3 =$$

$$9 \times 1 =$$

$$8 \times 9 =$$

$$9 \times 9 =$$

$$9 \times 3 =$$

$$9 \times 0 =$$

$$9 \times 9 = 9 \times 3 = 9 \times 0 = 9 \times 5 = 4 \times 9 =$$

$$4 \times 9 =$$

<u>x4</u>

<u>x5</u>

$$9 \times 2 =$$

$$2 \times 1 =$$

$$9 \times 2 = 2 \times 1 = 4 \times 9 =$$

$$5 \times 5 = 6 \times 2 =$$

$$8 \times 1 =$$

$$5 \times 6 =$$

$$8 \times 2 =$$

$$8 \times 1 = 5 \times 6 = 8 \times 2 = 9 \times 1 = 6 \times 0 =$$

Nines Strategy

Answer Key

9 x 2 = 18	9 x 7 = 63	9 x 3 = 27	9 x 1 = 9	8 x 9 = 72
9 x 9 = 81	9 x 3 = 27	9 x 0 = 0	9 x 5 = 45	4 x 9 = 36

9	2	1	6	9	7	9
<u>×6</u>	<u>x9</u>	<u>x5</u>	<u>x9</u>	<u>x3</u>	<u>x9</u>	<u>×8</u>
54	18	5	54	27	63	72
9	9	8	9	1	4	9
<u>×9</u>	<u>x4</u>	<u>x9</u>	<u>x7</u>	<u>x9</u>	<u>x9</u>	<u>x9</u>
81	36	72	63	9	36	81
2	9	8	1	7	9	9
<u>x4</u>	<u>x5</u>	<u>x9</u>	<u>x9</u>	<u>x9</u>	<u>x6</u>	<u>x0</u>
8	45	72	9	63	54	0

9 x 2 = 18	2 x 1 = 2	4 x 9 = 36	5 x 5 = 25	6 x 2 = 12
8 × 1 = 8	5 x 6 = 30	8 x 2 = 16	9 x 1 = 9	6 x 0 = 0

Squared Numbers Fact Strategy

Students will be working on the facts that are considered to be "square" numbers. (Example: $4^2 = 4 \times 4$. The visual image of this fact would be a 4 x 4 square.)

Materials Needed:

- Multiplication chart and student copies
- Colored tiles (100 per pair in baggies)
- Grid paper (per student)
- "Lovin' Math Facts" (per student)
- "Partner Practice" –Bristol
- Deck of Cards or 10-sided die (pair)

Teaching Fact Activity

Materials: grid paper, marker, 100 color tiles bagged for each pair of students, and sheets of grid paper.

Directions:

Let's take a look at a problem and talk about how we might solve it.

Carla lays tile floors. If the floor is 6 squares wide and 6 squares long, how many square tiles will it take to cover the floor? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Carla's bathroom floor is a square that is 4 tiles wide. How many tiles will it take to cover the floor? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Let's build some squares of different sizes using the color tiles. (*Demonstrate by building a 2 x 2 square with color tiles to ensure understanding of the two-dimensional square.*) Build as many different squares as you can with your color tiles in the next few minutes.

Describe your different squares (such as; 2 by 2, 2 groups of 2, 2 rows and 2 columns of 2). Let's draw the squares you have built with the tiles on the grid paper. Model some on the overhead. (Draw a couple of examples, making sure to label the dimensions of the squares.)

Now draw some of your squares, labeling their dimensions as you go. (Make the connection that these squares' dimensions are the factors and products for "square" numbers. Show how it would look to write a square number equation: $4^2 = 4 \times 4$; $4 \times 4 = 16$; $4^2 = 16$.)

Let's look at our multiplication chart. Color in the facts that are the squared numbers. What pattern do you see? (they make a diagonal through the multiplication chart.) Emphasize the relationships of the products on either side of the diagonal. For example, find the product for 4×8 and the product for 8×4 – noting their locations on the chart in reference to the diagonal of square products. Give students an opportunity to explore if this is true for all of the other facts on the chart. Students should discover that if they know the facts on one side of the diagonal of square products, then they know the facts

on the other side as well. These products represent the *commutative property* for multiplication.

Practice Fact Activity

Lovin' Math Facts

Materials: a deck of cards (with the face cards and 10s removed) or a 10-sided die, Lovin' Math Facts sheet for each child

The drill command will be to square the number that is rolled on the die or drawn from the deck of cards. Demonstrate a couple of examples. The directions are below:

Directions:

On the back of the *Lovin' Math Facts* sheet, square each number (0-9) and circle the products of all the squared numbers (i.e. $0 \times 0 = 0$) $1 \times 1 = 1$) $2 \times 2 = 4$ $3 \times 3 = 9$. .). Then, turn your paper over and write each product (circled numbers) in each of the stars. You will need to repeat some products until all stars are filled. As a number is generated from the die or the deck of cards, square that number (multiply it by itself) and cross off a star with its product. Cross off only one star at a time. If all stars are crossed off for a number, wait for the next number. Once all stars are crossed off, call out "I'm Lovin' Math Facts!"

Practice Fact Activity

Partner Practice Page

Materials: Square numbers practice partner page (Bristol)

Directions:

Use this page to practice the squared numbers. There will be review problems from previous learned strategies. Two people use this page. The person who is going to be the "coach" will hold the paper up between them so that the facts without the answers are facing the person who is the "student". The "student" should read the problem and then give the answer. The "coach" gives feedback as to whether the answer is right or wrong. Demonstrate positive responses to correct answers, such as..."Good job", "Alright", "Way to go", etc.)

Square Numbers Strategy

$$7 \times 7 =$$

$$7 \times 7 = 2 \times 2 =$$

$$6 \times 6 = 1 \times 1 =$$

$$1 \times 1 =$$

$$9 \times 9 =$$

$$3 \times 3 =$$

$$9 \times 9 = 3 \times 3 = 6 \times 6 = 5 \times 5 = 8 \times 8 =$$

$$5 \times 5 =$$

$$8 \times 8 =$$

2

<u>x8</u>

<u>x5</u>

<u>x5</u>

<u>×6</u>

$$7 \times 7 =$$

$$7 \times 7 = 4 \times 4 =$$

$$6 \times 5 =$$

$$6 \times 5 = 8 \times 8 =$$

$$2 \times 2 =$$

$$3 \times 3 -$$

$$5 \times 5 =$$

$$2 \times 2 = 3 \times 3 = 6 \times 5 = 5 \times 5 = 7 \times 7 =$$

Square Numbers Strategy

Answer Key

4 × 4 = 16	7 x 7 = 49	2 x 2 = 4	6 x 6 = 36	1 × 1 = 1
9 x 9 = 81	3 x 3 = 9	6 x 6 = 36	5 x 5 = 25	8 x 8 = 64

6	8	5	6	3	7	2
<u>×6</u>	<u>×8</u>	<u>x5</u>	<u>x6</u>	<u>x3</u>	<u>×7</u>	<u>x2</u>
36	64	25	36	9	49	4
9	7	3	2	8	4	5
<u>×9</u>	<u>×7</u>	<u>x3</u>	<u>×2</u>	<u>×8</u>	<u>×4</u>	<u>x5</u>
81	49	9	4	64	16	25
1	6	8	5	5	5	5
<u>×5</u>	<u>x5</u>	<u>×5</u>	<u>x0</u>	<u>x7</u>	<u>x6</u>	<u>x4</u>
5	30	40	0	35	30	20

4 × 1 = 4	7 x 7 = 49	4 × 4 = 16	6 x 5 = 30	8 x 8 = 64
2 x 2 = 4	3 x 3 = 9	6 x 5 = 30	5 x 5 = 25	7 x 7 = 49

Grid Paper

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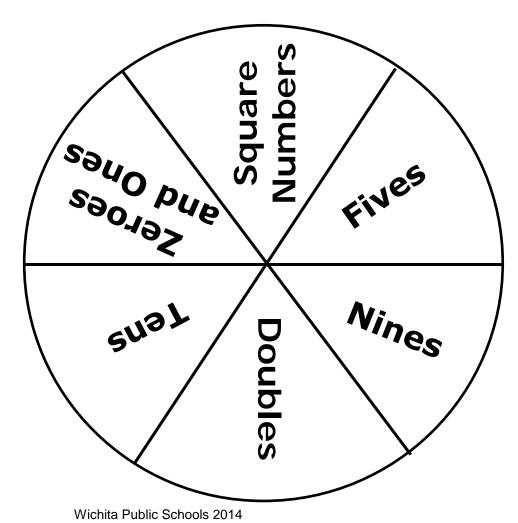
Strategy Focus Review

Students need to practice the strategies that have been taught. They need to start reasoning out which facts are best solved with particular strategies. This should be intentionally practiced and discussed.

Use the spinner below with the facts on the next page. Give each student a copy of the facts page. Place the spinner under the document camera and spin it. The spinner will indicate which strategy the students will focus on for their facts page. If the spinner lands on "Doubles," then the students will identify the facts on their page that can easily be solved using that strategy and circle them.

After a short amount of time, place a copy of the facts page under the document camera and have the students tell you which facts were circled. Have students explain why that fact can be solved with that strategy for the first 3 or 4 given to you.

When facts have been circled and verified that they can be solved using that particular strategy, have the students answer the problems that are circled using the indicated strategy. Have students pair up when finished to check their answers to see if they agree.



Strategy Focus Review (Doubles, Fives, Zeroes and Ones, Tens, Nines, Square Numbers)

4	9	0	10	1
<u>x 4</u>	<u>x 3</u>	<u>x 8</u>	<u>x 7</u>	<u>x 6</u>
7	9	1	9	8
<u>x 2</u>	<u>x 9</u>	<u>x 4</u>	<u>x 2</u>	<u>x 5</u>
3	6	10	8	6
<u>x 3</u>	<u>x 2</u>	<u>x 4</u>	<u>x 5</u>	<u>x 0</u>
1	5	3	5	6
<u>x 7</u>	<u>x 2</u>	<u>x 9</u>	<u>x 5</u>	<u>x 10</u>
4	1	4	7	9
<u>x 9</u>	<u>x 3</u>	<u>x 2</u>	<u>x 7</u>	<u>x 5</u>

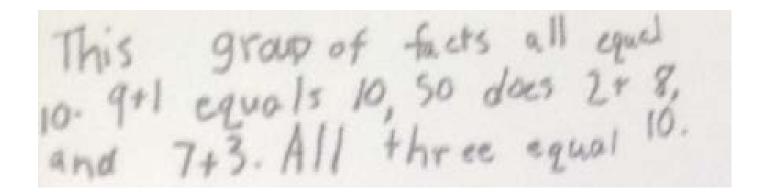
Strategy Focus Review

Students need to be able to discuss and write about their reasoning with strategies. This activity continues the focus on strategies but engages the students cognitively by writing about their reasoning for a particular strategy for a specific fact or set of facts.

Use the previous page (pg. 60) to create a set of cards for each student or pair of students. These cards will be sorted on the strategy mat on the following page. After the cards have been sorted, the students will check their thinking with a partner and then discuss any discrepancies between them. The students need to explain to their partner why they choose that particular strategy that may be different from their partner. This doesn't mean their strategy use was incorrect (because one fact could be solved by more than one strategy) but listening to another's reasoning could let them see another way to attack problems.

Students may switch facts around while they are talking with their partners or they may decide to leave them. The facts are finally written, taped or pasted onto the chart (if students are taping or pasting you will want to enlarge the chart onto 11 x 17 paper to allow for the cards and the writing at the bottom) and then one strategy is chosen to explain why those facts where chosen to be solved with that strategy.

Here is a student example of their reasoning – although this is with an addition strategy, you can see the thinking the student went through in using a strategy to solve the facts.



Strategy Sort Mat

Tens	Nines	Squared Numbers
Doubles	Zeroes and Ones	Fives
Choose one strategy and ex	plain why those facts are solve	d with the chosen strategy:

Using Known Facts to Derive Other Facts

Only 25 multiplication facts remain (actually fewer, due to the commutative property). These facts can be learned by relating each to an already known fact (or helping fact). For example, 4 x 6 is connected to the Doubles strategy by doubling and then doubling again. 6 x 7 can be solved by reasoning that it is just one group of 7 away from 5 x 7 which is 35 so 7 more makes 42.

The rest of the strategies in this book are the facts that are built by using the strategies the students have already learned. The reasoning and relationship building that the students will be using for the rest of the strategies is critical. They need to be able to discuss their thinking with their peers and with you.

Double and Double Again Strategy

Students will be working on the facts that have <u>4 as one of the factors</u>. This strategy builds upon the **Doubles** Strategy. Students look at the other factor and double it, then double that answer. (Example: 4 x 7. Double 7 to get 14. Double 14 to get 28.)

Materials Needed:

- Multiplication chart and student copies (use copy from Day 1)
- "Slide Rule" page (each)
- 5 sheets of blank copy paper (each)
- Partner Practice Page (pair)

Focus

Directions:

Let's take a look at a problem and talk about how we might solve it.

Treats come in bags of 8. If there are 4 bags, how many treats are there? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Mark is at the game room and has put his last \$6 into the token machine. He gets four tokens for every dollar. How many tokens did he receive? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

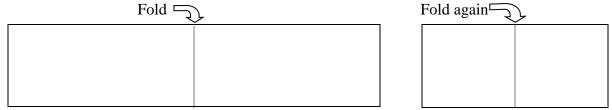
Teaching Fact Activity

Materials: 5 blank sheets of paper-cut into a 4 ¼ inch by 11 inch strips (to demonstrate), 5 blank papers per student, markers

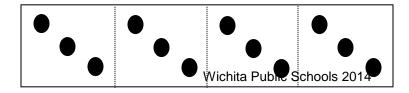
Let's review the <u>Doubles</u> strategy. (Remind them that the <u>Doubles</u> are the "x 2" facts.) Today we will continue with the 4s.

When you are working with the facts that have $\underline{4}$ as one of the factors, you can use the <u>Double and Double Again</u> strategy. For this strategy, you double the other factor, then double that answer to get the final answer for the problem. (Example: 4×3)

Let's use the strip of paper that you have in front of you to make "Snap It" cards. Fold the paper in half and then in half again. Example:



Unfold the paper and make three dots on each section of the paper. This represents the fact 4×3 .



Fold the paper up again so that you see one of the sets of dots.



To "double" it, open the paper up (it's more effective if you make the paper *Snap* when you do it). So 3 doubled is 6. Then 6 doubled (open the paper up again with a *Snap*) is 12. This is <u>Double and Double Again!</u> (Do this with the rest of the x4 facts. Spend time practicing the x4 facts with the "Snap It" cards.)

Practice Fact Activity

Materials: Slide Rule page for each student

Give each child a Slide Rule page

Directions:

Another fact activity is the **Slide Rule**. Take your paper and fold it along the first dotted line. (Show with a sample of your own.)

Show the students the side with the problem while you look at the answer side. Have students say the problem aloud and have them follow the "thinking" part below the problem. You can also have the student fold back the paper and write the answers on another piece of paper. This allows for easy checking of answers.

Practice Fact Activity

Partner Practice Page

Materials: Double and double again practice partner page (Bristol)

Directions:

Use this page to practice the "x 4s". Review problems from previously learned strategies will be included. Two people use this page. The person who is going to be the "coach" will hold the paper up between them so that the facts without the answers are facing the person who is the "student". The "student" should read the problem and then give the answer. The "coach" gives feedback as to whether the answer is right or wrong.

Slide Rule

Double and Double Again Strategy

			- ,
4 x 6 = 6 d	24	7 x 4 =	28
Think 6 + 6 = 12 Then 12 + 12 = 24		Think 7 doubled is 14. Then 14 doubled is ?	
8 x 4 =	32	3 x 4 =	12
Think 8 + 8 = 16 Then 16 + 16 = ?		Think 3 doubled is 6. Then 6 doubled is ?	
4 x 3 =	12	4 x 8 =	32
Think 3 + 3 = ? Then		Think 8 doubled is ? Then	
7 x 4 =	28	4 X 6 =	24
Think 7 + 7 = ? Then		Think Then	
6 x 4=	24	4 × 7 =	28
Think 6 + 6 = ? Then		Think Then	
4 x 8 =	32	8 × 4 =	32
Think 8 + 8 = ? Then			
8 x 4=	32	4 x 3 =	12
Think 8 + 8 = ? Then			
3 x 4=	12	6 x 4 =	24
Think 3 + 3 = ? Then			

Double and Double Again Strategy









$$4 \times 3 = 4 \times 6 =$$

$$4 \times 6 =$$

$$4 \times 7 = 4 \times 6 =$$

$$4 \times 6 =$$

$$8 \times 4 =$$

$$3 \times 4 =$$

$$6 \times 4 =$$

$$3 \times 4 =$$

$$8 \times 4 = 3 \times 4 = 6 \times 4 = 3 \times 4 = 8 \times 4 =$$

<u>×4</u>

4

4

4

8

4

<u>x4</u>

<u>×4</u>

<u>x3</u>

<u>×6</u>

<u>x4</u>

<u>x7</u>

<u>8×</u>

4

<u>x3</u>

8 <u>x5</u>

4 <u>x3</u>

7 <u>x4</u>

2

<u>x6</u>

<u>x8</u>

$$7 \times 7 =$$

$$6 \times 6 = 4 \times 3 = 7 \times 7 = 6 \times 4 = 8 \times 4 =$$

$$8 \times 4 =$$

$$7 \times 4 =$$

$$3 \times 3 =$$

$$2 \times 8 =$$

$$7 \times 4 = 3 \times 3 = 2 \times 8 = 4 \times 7 = 4 \times 6 =$$

Double and Double Again Strategy

Answer Key

4 × 3 = 12	4 x 6 = 24	4 x 8 = 32	4 x 7 = 28	4 × 6 = 24
8 x 4 = 32	3 x 4 = 12	6 x 4 = 24	3 x 4 = 12	8 x 4 = 32

6	8	3	6	3	7	6
<u>x4</u>	<u>x4</u>	<u>×4</u>	<u>x4</u>	<u>×4</u>	<u>x4</u>	<u>x4</u>
24	32	12	24	12	28	24
4	7	4	4	8	4	4
<u>x8</u>	<u>x4</u>	<u>x3</u>	<u>x6</u>	<u>x4</u>	<u>x7</u>	<u>×8</u>
32	28	12	24	32	28	32
4	6	8	4	7	2	4
<u>x3</u>	<u>x4</u>	<u>×5</u>	<u>x3</u>	<u>x4</u>	<u>x6</u>	<u>×8</u>
12	24	40	12	28	12	32

6 x 6 = 36	4 × 3 = 12	7 x 7 = 49	6 x 4 = 24	8 x 4 = 32
7 x 4 = 28	3 x 3 = 9	2 x 8 = 16	4 x 7 = 28	4 x 6 = 24

Double and One More Set Strategy

Facts to focus on: 3 x 6, 3 x 7, 3 x 8, 6 x 3, 7 x 3, 8 x 3

Materials Needed:

- Multiplication chart and student copies
- Arrays for focused facts (can create additional arrays for x3 facts)
- Graph paper
- Scissors
- Dice and Deck of cards or 10-sided die
- Drill Command Worksheet
- Partner Practice Page (pair) (Bristol-copy back to back)

Focus:

Materials: multiplication chart, colored pencil

Let's review the **Doubles** strategy from the first week. We will now continue with the remaining \mathbf{x} 3s (3 x 6, 3 x 7, 3 x 8).

Let's take a look at a problem and talk about how we might solve it.

Kevin found 7 pennies. Fred found 3 times as many pennies as Kevin. How many pennies did Fred find? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Bill is going to see his grandparents in 3 weeks? How many days until he sees them? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols ~ help students recall that there are 7 days not 5 in a week.)

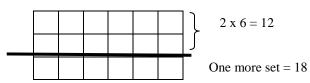
Teaching Fact Activity:

Materials: Students will need graph paper to create the remaining x3 facts. There is a sample of x3 Arrays to demonstrate the three remaining x3 facts.

When you are working with the facts that have **3** as one of the factors, you can use the **Double and One More Set** strategy. For this strategy you will recall the double and add on one more set of the number doubled.

Use graph paper to cut out the arrays for 3×6 , 3×7 , and 3×8 . Draw a line where the doubles array would be (i.e. $2 \times 6 = 12$) and point out the additional set that needs to be added on (one more set of 6 = 18).

Example:



This is an example of using the Distributive Property to solve these x3 problems.

For example:
$$6 \times 3 = 6 \times (2 + 1)$$
.
 $6 \times 3 = (6 \times 2) + (6 \times 1)$
 $6 \times 3 = 12 + 6$

Practice Fact Activity:

Materials: a deck of cards (with the face cards and 10s removed) or a 10-sided die, **Drill Command** worksheet, dry erase markers and eraser. Have each child mark drill commands at the top of each column. The first column should be x3. The remaining columns can be any command they would like to have extra practice on. Use the copy to model your expectations.

As you generate a number from your deck of cards (or 10-sided die), think about what the product (answer) will be when the drill command is used with the number. For example: when the drill command is "x 3" and I see the number "8", I think 8 x 2 = 16 then 16 + 8 = 24, so 8 x 3 has the answer "24". It is then recorded in the "x 3" column. Continue until all slots are filled on the **Drill Command** worksheet.

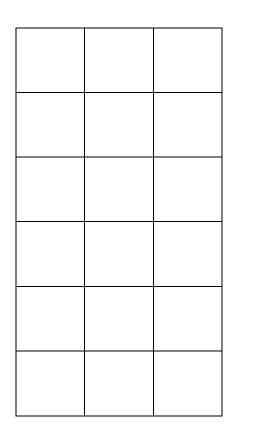
Practice Fact Activity:

Partner Practice Page

Materials: Double and one more set partner practice page (Bristol)

Use this page to practice the "x 3s". Review problems from previously learned strategies will be included. Two people use this page. The person who is going to be the "coach" will hold the paper up between them so that the facts without the answers are facing the person who is the "student". The "student" should read the problem and then give the answer. The "coach" gives feedback as to whether the answer is right or wrong.

x 3 Arrays





Double and One More Set Strategy

$$4 \times 3 = 3 \times 6 = 3 \times 8 = 3 \times 7 = 3 \times 2 =$$

$$8 \times 3 = 3 \times 3 = 6 \times 3 = 3 \times 3 = 9 \times 3 =$$

$$6 \times 6 = 3 \times 3 = 3 \times 1 = 6 \times 4 = 8 \times 2 =$$

$$7 \times 5 = 3 \times 9 = 3 \times 8 = 0 \times 7 = 3 \times 6 =$$

Partner Practice Page

Double and One More Set Strategy

Answer Key

4 × 3 = 12	3 x 6 = 18	3 x 8 = 24	3 x 7 = 21	3 x 2 = 6
8 x 3 = 24	3 x 3 = 9	6 x 3 = 18	3 x 3 = 9	9 x 3 = 27

6	8	3	6	3	7	6
<u>×3</u>	<u>x3</u>	<u>x3</u>	<u>×3</u>	<u>x3</u>	<u>x3</u>	<u>×3</u>
18	24	9	18	9	21	18
3	3	4	3	8	3	3
<u>×9</u>	<u>x4</u>	<u>x3</u>	<u>×6</u>	<u>x3</u>	<u>x2</u>	<u>x8</u>
27	12	12	18	24	6	24
4	6	7	4	5	3	3
<u>×4</u>	<u>x3</u>	<u>×7</u>	<u>×3</u>	<u>×3</u>	<u>×6</u>	<u>x8</u>
16	18	49	12	15	18	24

6 × 6 = 36	3 x 3 = 9	3 x 1 = 3	6 x 4 = 24	8 x 2 = 16
7 × 5 = 35	3 x 9 = 27	3 x 8 = 24	0 x 7 = 0	3 x 6 = 18

Half then Double Strategy

Facts to focus on: 6 x 7, 6 x 8, 7 x 8, 7 x 6, 8 x 6, 8 x 7

Materials Needed:

- Multiplication chart wall chart and student copies
- Array Finder model and student copies
- Drill Doughnuts page
- · Coffee stirrers or Wikki Stix

Focus:

Let's take a look at a problem and talk about how we might solve it.

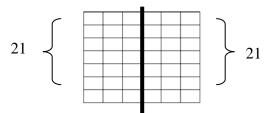
Paul likes to buy eggs in half-dozen containers. He has 7 full containers in his refrigerator. How many eggs? (Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Hamburger buns come 8 to a package. Seven packages will make how many hamburgers? Students respond and explain their thinking using words, pictures, manipulatives, or symbols.)

Teaching Fact Activity:

Materials: Array Finder, coffee stirrer or Wikki Stix

When working with problems that have an even factor, you can use this strategy. Let's use the array finder to show the fact "6 x 7". Use the copy to show the students how to move over 6 spaces and down 7 spaces. This will show an array of 42 squares. In our problem, 6 x 7, which factor is an even number? (6) What is **half** of 6? (3) Take your coffee stirrer and place it at the halfway mark (3 x 7 on one half, 3 x 7 on the other half). Notice that the problem "6 x 7" has been cut in half into a smaller fact we have already learned—3 x 7 = 21. Now we double 21 (21 + 21) to get the answer ... 42! This is the Half and then Double Strategy:

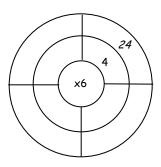


Have students find a partner. They should each share a different problem to show how this strategy works. You may wish to write the "Facts to Focus On" (found under strategy title) on the overhead or chalkboard.

Practice Fact Activity:

Materials: Drill Doughnut worksheet, pencil

To practice this fact strategy, we are going to do an activity called Drill Doughnuts. The drill command is in the center of the doughnut. Work your way to the outside by multiplying the number printed by the drill command. For example; if x6 is in the center of the doughnut and the number 4 is printed next to it, multiply 6 by the 4 and write the product 24 along the edge of the doughnut beside the 4. The fourth doughnut is to practice a fact of your choice. (The blank worksheet is to create your own for any facts practice.)



Practice Fact Activity

Partner Practice Page

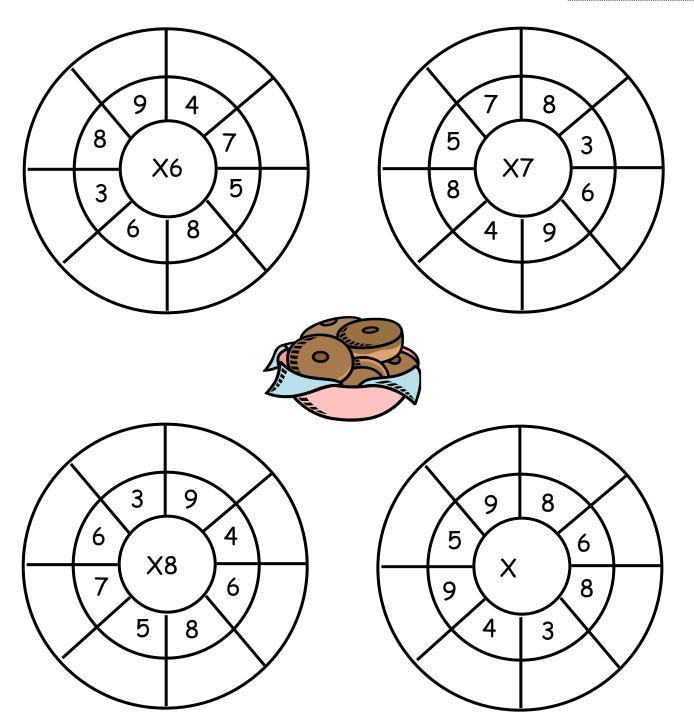
Materials: Half then double practice partner page (Bristol)

Use this page to practice all strategies. Review problems from previously learned strategies. Two people use this page. The person who is going to be the "coach" will hold the paper up between them so that the facts without the answers are facing the person who is the "student". The "student" should read the problem and then give the answer. The "coach" gives feedback as to whether the answer is right or wrong.



Drill Doughnuts

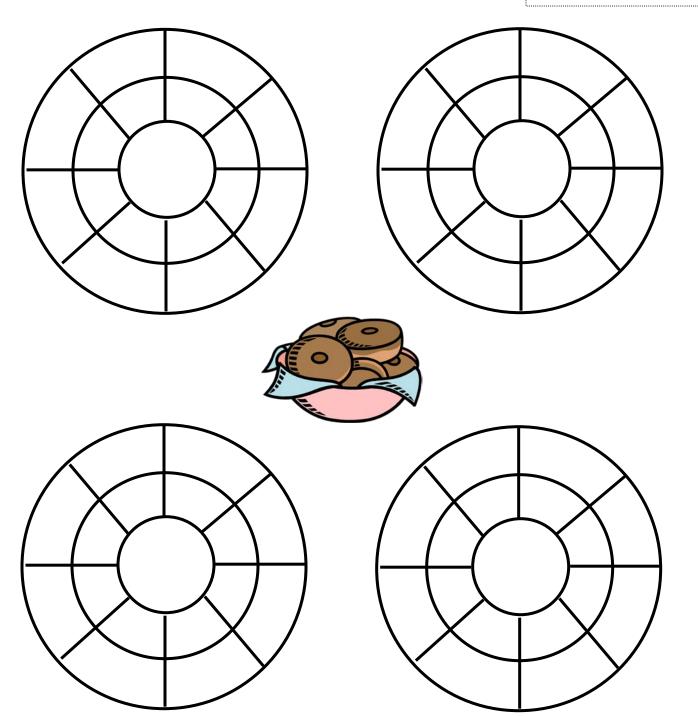
Name





Drill Doughnuts

lama





Partner Practice Page

Half Then Double Strategy

$$4 \times 6 = 0 \times 6 = 5 \times 8 = 8 \times 6 = 8 \times 5 =$$

$$8 \times 4 = 3 \times 8 = 6 \times 9 = 8 \times 3 = 9 \times 6 =$$

$$6 \times 6 = 3 \times 3 = 3 \times 1 = 6 \times 4 = 8 \times 2 =$$

$$7 \times 7 = 3 \times 9 = 3 \times 0 = 8 \times 7 = 7 \times 6 =$$

Partner Practice Page

Half Then Double Strategy

Answer Key

4 × 6 = 24	0 x 6 = 0	5 x 8 = 40	8 x 6 = 48	8 x 5 = 40
8 x 4 = 32	3 x 8 = 24	6 x 9 = 54	8 x 3 = 24	9 x 6 = 54

6	8	2	6	8	7	6
<u>×8</u>	<u>x6</u>	<u>x6</u>	<u>x3</u>	<u>x3</u>	<u>x6</u>	<u>×6</u>
48	48	12	18	24	42	36
4	4	6	8	8	6	5
<u>x8</u>	<u>x6</u>	<u>x3</u>	<u>x6</u>	<u>x7</u>	<u>x8</u>	<u>×8</u>
32	24	18	48	56	48	40
4	6	7	7	5	3	9
<u>×4</u>	<u>x3</u>	<u>×7</u>	<u>×9</u>	<u>x3</u>	<u>x7</u>	<u>x2</u>
16	18	49	63	15	21	18

6 x 6 = 36	3 x 3 = 9	3 × 1 = 3	6 x 4 = 24	8 × 2 = 16
7 x 7 = 49	3 x 9 = 27	3 x 0 = 0	8 x 7 = 56	7 × 6 = 42

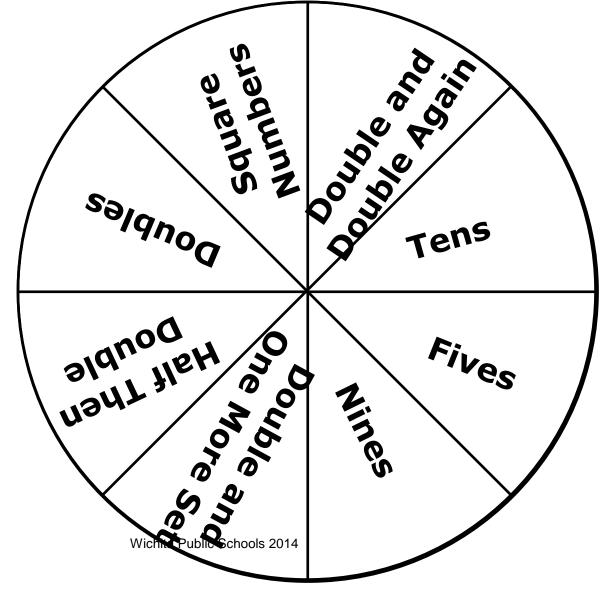
Strategy Focus Review

Students need to practice the strategies that have been taught. They need to start reasoning out which facts are best solved with particular strategies. This should be intentionally practiced and discussed.

Use the spinner below with the facts on the next page. Give each student a copy of the facts page. Place the spinner under the document camera and spin it. The spinner will indicate which strategy the students will focus on for their facts page. If the spinner lands on "Doubles," then the students will identify the facts on their page that can easily be solved using that strategy and circle them.

After a short amount of time, place a copy of the facts page under the document camera and have the students tell you which facts were circled. Have students explain why that fact can be solved with that strategy for the first 3 or 4 given to you.

When facts have been circled and verified that they can be solved using that particular strategy, have the students answer the problems that are circled using the indicated strategy. Have students pair up when finished to check their answers to see if they agree.



80

Strategy Focus Review (Doubles, Fives, Zeroes and Ones, Tens, Nines, Square Numbers, Double and Double

Again, Double and One More Set, Half Then Double)

	Agairi, Double air	a One More Set, I	Haif Then Double)	
4	9	2	10	8
<u>x 7</u>	<u>x 3</u>	<u>x 8</u>	<u>x 7</u>	<u>x 6</u>
7	9	5	9	8
<u>x 2</u>	<u>x 9</u>	<u>x 4</u>	<u>x 2</u>	<u>x 5</u>
3	6	10	8	6
<u>x 3</u>	<u>x 2</u>	<u>x 4</u>	<u>x 6</u>	<u>x 5</u>
3	8	3	6	3
<u>x 7</u>	<u>x 7</u>	<u>x 9</u>	<u>x 7</u>	<u>x 6</u>
4	6	4	7	9
<u>x 9</u>	<u>x 6</u>	<u>x 2</u>	<u>x 7</u>	<u>x 5</u>

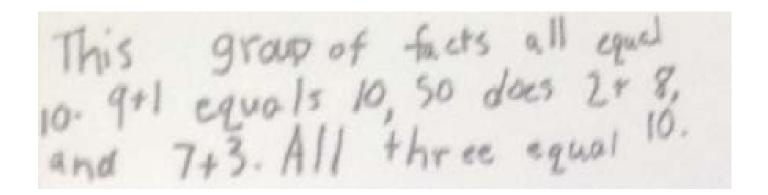
Strategy Focus Review

Students need to be able to discuss and write about their reasoning with strategies. This activity continues the focus on strategies but engages the students cognitively by writing about their reasoning for a particular strategy for a specific fact or set of facts.

Use the previous page (pg. 81) to create a set of cards for each student or pair of students. These cards will be sorted on the strategy mat on the following page. After the cards have been sorted, the students will check their thinking with a partner and then discuss any discrepancies between them. The students need to explain to their partner why they choose that particular strategy that may be different from their partner. This doesn't mean their strategy use was incorrect (because one fact could be solved by more than one strategy) but listening to another's reasoning could let them see another way to attack problems.

Students may switch facts around while they are talking with their partners or they may decide to leave them. The facts are finally written, taped or pasted onto the chart (if students are taping or pasting you will want to enlarge the chart onto 11 x 17 paper to allow for the cards and the writing at the bottom) and then one strategy is chosen to explain why those facts where chosen to be solved with that strategy.

Here is a student example of their reasoning – although this is with an addition strategy, you can see the thinking the student went through in using a strategy to solve the facts.



Strategy Sort Mat

Doubles	Fives	Tens	Nines
Squared	Double and	Half Then	Double and One
Numbers	Double Again	Double	More Set
 Choose one strategy a	l and explain why those f	l facts are solved with th	l ne chosen strategy:

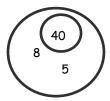
Multiplication/Division Related Equations

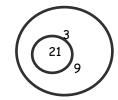
Teaching Fact Activity:

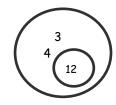
Materials: white/chalk board

Show students, without explanation, families of numbers with the product circled.

Example:







<u>Ask why they think the numbers go together</u> and why one number is circled. Record the number sentences that represent the relationship of the numbers within the Venn diagram.

Example:
$$8 \times 5 = 40$$

 $5 \times 8 = 40$

$$40 = 8 \times 5$$

 $40 = 5 \times 8$
 $40 \div 5 = 8$

$$8 = 40 \div 5$$

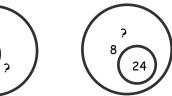
 $5 = 40 \div 8$

$$40 \div 3 = 8$$

 $40 \div 8 = 5$

When this number family idea is fairly well understood, show some families with one number replaced by a question mark. For example:





Ask what number is missing. When students understand this activity, explain that you have missing-number worksheets based on this idea. Each card has two or three numbers that go together in the same way. Sometimes the circled number is missing (the product), and sometimes one of the other numbers is missing (a factor). The object is to name the missing number.

Practice Fact Activity:

"Missing-Number Worksheets" (<u>Teaching Student-Centered Mathematics</u> vol. 2, Activity 3.6) **Materials:** missing-number worksheet(s), answer key

Directions:

- 1. Have students fill in the missing numbers on the worksheet. The facts have been listed according to their fact strategy.
- 2. Because students do not always connect this knowledge of the missing part to multiplication or division, it is important that students write multiplication and division problems.
- 3. You may wish to give one column to focus on each strategy as it's learned or review strategies that students are comfortable with. You may also differentiate according to the strategies your students need additional practice.
- 4. Answer keys are also included. These could be a nice set of miniature flash cards students could use to practice at home. ©

Practice Fact Activity:

"Triangle Flash Cards"

Materials: flash cards (each)

Triangle flash cards can be used individually, as well as in small group settings.

Directions:

- 1. To practice multiplication facts, hold the triangle so that your thumb covers the product (the underlined number).
- 2. Multiply the two factors shown to identify the product that is covered.
- 3. To practice division facts, cover one of the **factors** (instead of the product).

It is best to focus on a smaller set of facts at a time. This can be done by focusing on a specific strategy, or by a chosen set of problems that are considered "hard" for the student.

Doubles

Zeroes/Ones/Fives

Tens

Double and One More Set

Half and Then Double

Double and One More Set	Haif and Then Double	
6 3	7 6	
7 (21)	48 8	
8 (24)	8 8	
3 (27)	56 8	
4 3	8 6	
5 3	6 54	
3 7	8 56	
9 (27)	6 6	
3 (18)	30 6	
24 3	6 7	
3 (6)	8 4	
0 3	6 4	

40

Answer Key

Doubles

Zeroes/Ones/Fives

Tens

Missing Number Worksheet Answer Key

Squared

Nines

Double and Double again

36 6 6

9 5 (45)

4 (8) 2

4 4 (16)

18 2 9

4 (12) 3

7 (49) 7

6 9 (54)

24) 6 4

3 (9) 3

72 8 9

8 4 (32)

6 6 36)

9 (27) 3

20 5 4

9 (81) 9

4 9 (36)

28) 4 7

(4) 2 2

9 7 (63)

4 (36) 9

7 7 (49)

9 1 9

12 3 4

8 (64) 8

54) 6 9

4 (32) 8

(1) 1 1

9 (72) 8

7 4 (28)

8 8 64)

2 (18) 9

8 2 4

5 (25) 5

9 3 (27)

6 4 24

81) 9 9

63 7 9

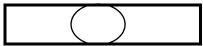
9 (36) 4

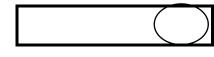
Answer Key

Double and One More Set

Half and Then Double

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Practice and Review

Students now have at least eight strategies that can help them solve multiplication facts. The important part now is to practice using these strategies. You may choose to repeat any activities that have been previously covered that focus on a particular set of facts, or you may choose any of the following activities that will give students the opportunity to practice various strategies with numerous facts. Your decisions should be based upon your students' needs.



Random Number Generator

Materials: Drill Command sheets, random number generator (CD <u>or</u> spinner <u>or</u> 10-sided die)

About the Activity:

The random number CD is a valuable tool for motivating students with drill and practice. The random number CD is designed to generate the digits 0-9 with background music. This helps students learn to filter out unnecessary sounds and listen for important information. The first few songs are read at 8-second intervals, the last few are read at 5-second intervals.

The random number CD can be used for early learners to practice recording the numbers that are heard. This is an excellent assessment tool for auditory number recognition.

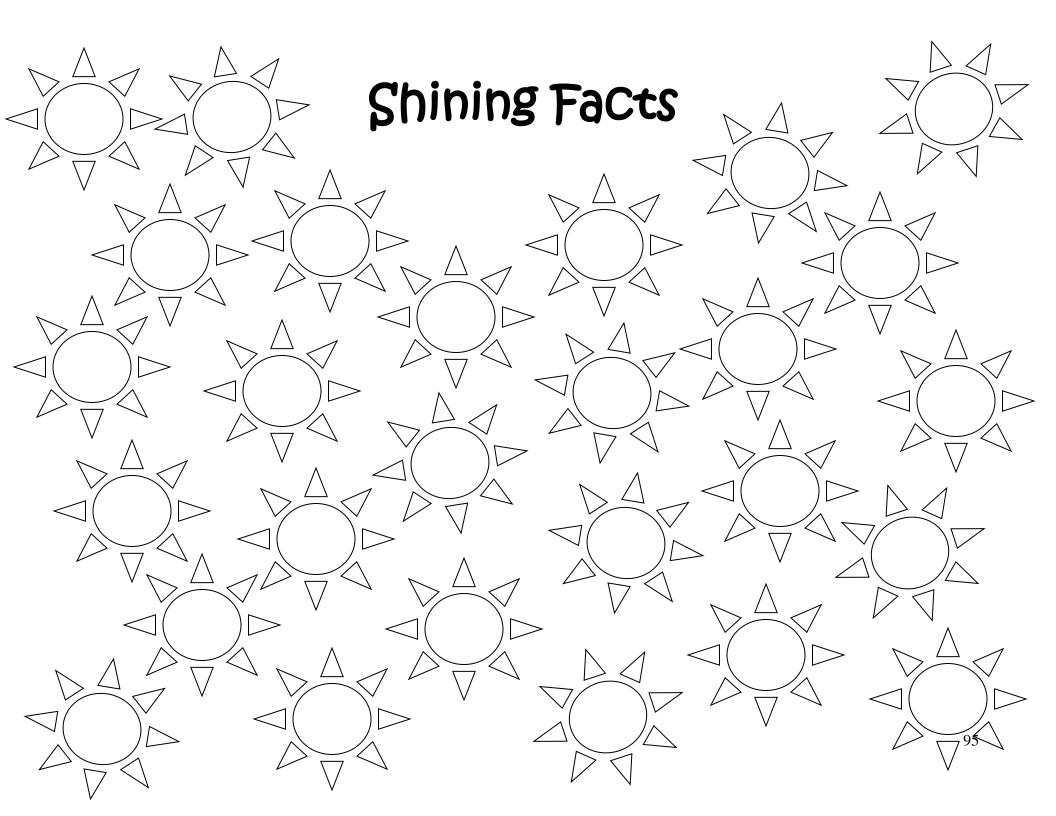
The random number CD can be used with any drill command for multiplication. For instance, if the drill command were x6, then students would be working on this set of basic multiplication facts:

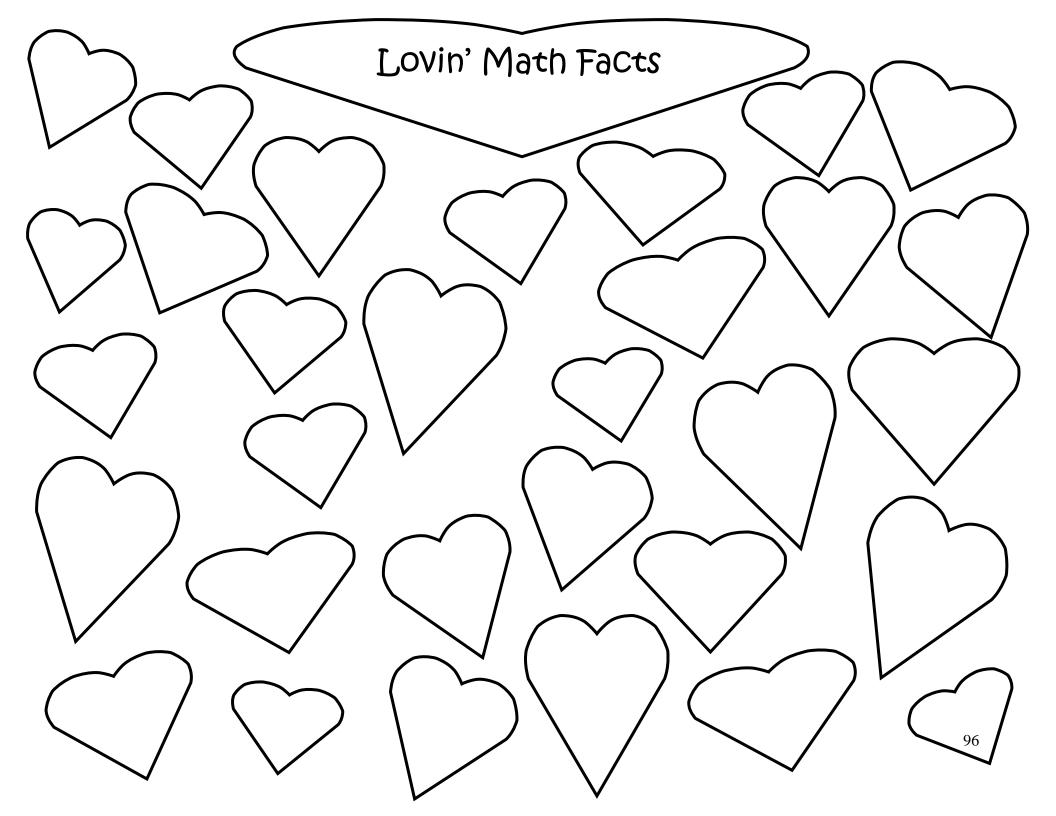
$0 \times 6 = 0$	$5 \times 6 = 30$
$1 \times 6 = 6$	$6 \times 6 = 36$
$2 \times 6 = 12$	$7 \times 6 = 42$
$3 \times 6 = 18$	$8 \times 6 = 48$
$4 \times 6 = 24$	$9 \times 6 = 54$

The random number CD can be used with grid paper and an identified drill command to assess the students' competency with any set of facts. The student would record the drill command at the top square of the grid paper. When a number is called out, the student would perform that drill command and record the sum or product. This would continue down the columns. Mixed drill commands could also be given for each column.

Adapted from K. Sutton materials

			Name Date
	Drill Command	d Practice Shee	†
Drill Command	Drill Command	Drill Command	Drill Command

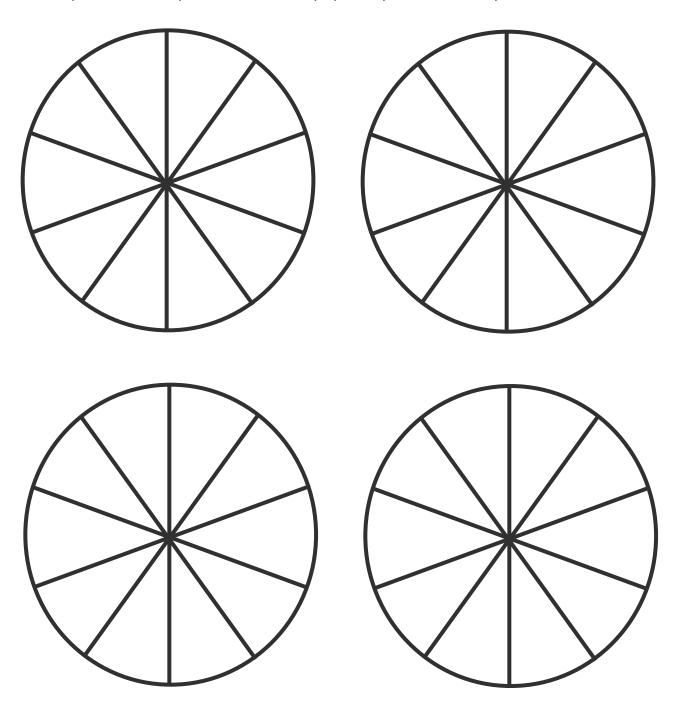




Spinners

Random Number Generator

Put numbers in the spaces of the spinner. Place a paper clip over the center of the spinner and the point of a pencil at one end of the paper clip and at the center point of the spinner. Flick the paper clip to make it spin.



Number Strips

Random Number Generator

Each student would have their own number strip to practice a specific strategy. The student will lay the number strip beside their paper and write the answer for the drill command beside it. Making larger versions for the chalkboard are also very motivational for students to practice their facts.

9
4
3
2
8
5
1
7
6
8

	-
5	
7	
3	
6	
4	
1	
9	
8	
2	
5	

3
6
7
4
9
8
5
1
2
8

7	
6	
4	
8	
2	
3	
5	
1	
9	
4	

1
6
8
7
6
2
3
4
8
9

"BEAT THE TEACH"

1 2 3 4 5 6 7 8 9

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

PLAYERS: Class vs. the teacher, class divided into 2 teams, small center teams, or 2 students

MATERIALS:

- One "Beat the Teach" game board.
- Two different markers (paper clips, chips, etc.) and a marker.

DIRECTIONS: The classroom challenges the teacher. A decision is made as to who has the first turn. That challenger chooses two numerals at the top of the game board. Two different markers are placed at these numerals. If the classroom is first, a student chooses two numerals, multiplies them together and that challenger's marker is put on the product. Use X's and O's. Only one marker can be moved at a time. This is where the strategy comes in. Turns are alternated between teacher and class responding. The first challenger to get three products in a row is the winner. The objective for the class is to "Beat the Teach."

Crack the Code

1	2	3
4	5	6
7	8	9

Multiplication

Materials:

- Game boards (made out of poster board or copy the attached blacklines onto Bristol paper and laminate)
- Two 10-sided die
- Connecting cubes to use for game pieces (or other marker)

Directions:

- 1. Two to three players may play on the same game board.
- 2. Give player one a set of 10 connecting cubes in one color. Player two receives a set of 10 cubes but in a different color. If there is a third player, they get a set of cubes in a different color from the other two players.
- 3. The first player rolls the two number cubes and multiplies them together.
- 4. If the problem is answered correctly, the player places one of his colored cubes onto that answer on the game board. (Ex.: 6 x 4 = 24, so a connecting cube is placed on the 24 on the board). If they answer the problem incorrectly, the player doesn't place a cube onto the board.
- 5. Play goes to the next player and continues the same way.
- 6. When a player rolls a problem whose answer is covered by another player's cube, the player that rolled can "Knock Their Block Off" by removing their opponent's cube and placing their own in its place.
- 7. The first player to get 5 of their connecting cubes in a horizontal, vertical, or diagonal row WINS!

9	25	48	49	72
81	42	16	40	36
63	56	28	0	45
42	64	32	24	30
54	27	28	35	42

42	64	49	42	25
16	32	45	36	63
72	28	20	35	12
56	32	21	40	54
24	18	28	30	48

42	24	63	64	14
32	45	48	49	42
35	20	56	81	28
72	16	25	6	28
12	28	30	40	54

35	8	16	28	64
42	36	40	12	6
20	45	36	32	24
25	54	63	24	32
30	O	42	81	72

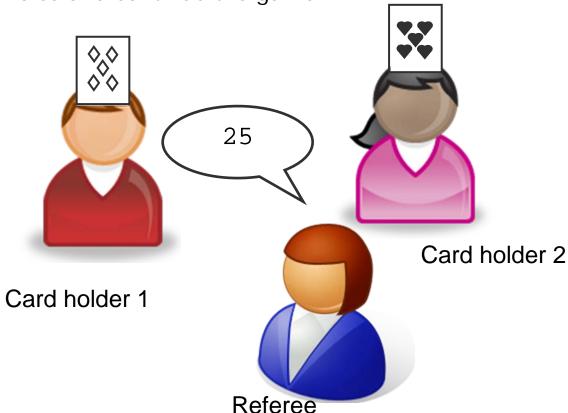
Salute!

Materials Needed: deck of cards (remove face cards—except one to represent zero, or numbers written on 5 x 7 cards (0 – 10)

Directions:

- 1. This activity needs 3 students to participate. One student is the referee, and the other 2 are the card holders. These 2 people face each other.
- 2. Each card holder draws a card from the deck without looking at the number on it. When the referee says, 'Salute,' each card holder places the card on his or her forehead (without looking).
- 3. The referee tells the 2 card holders the product (or quotient). By looking at the other person's card, each card holder should determine the value of their card.

4. Once both numbers are determined, students change roles and continue the game.



Math War

Players: 2

Materials: deck of playing cards or digit cards

Directions:

- 1. Deal out all the cards to each player. The players place the stack of cards face down in front of them.
- 2. When the dealer says "Go", the players will take the first 2 cards at the top of their stack and turn it face up next to their stack of cards.
- 3. All the players look at their cards and whoever has the largest product gets all of the cards that are face up.
- 4. If the players have cards that have the same product, then a war takes place. The players with the matching product take the next 2 cards and multiply those. Whoever has the largest product with the 2 new cards wins those cards plus the cards from the previous hand.

Optional version: Players will play for the smallest product instead. Whoever has the smallest product will win the cards.

