Decomposing and Composing Algorithms

Common Core Standards

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.NBT.6</td>
<td>Add up to four two-digit numbers using strategies based on place value and properties of operations.</td>
</tr>
<tr>
<td>2.NBT.7</td>
<td>Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</td>
</tr>
<tr>
<td>2.NBT.9</td>
<td>Explain why addition and subtraction strategies work, using place value and the properties of operations.</td>
</tr>
</tbody>
</table>

Having students work in the base-ten system contributes to students’ deeper understanding of numbers and their place value. In learning about whole numbers, students learn that ten ones compose a new kind of unit called a ten. They understand two-digit numbers as composed of tens and ones, and use this understanding in computations, decomposing 1 ten into 10 ones and composing a ten from 10 ones.

In this unit of study students learn to decompose numbers in at least two ways.

352

Possible responses:
- 3 hundreds + 5 tens + 2 ones
- 2 hundreds + 15 tens + 2 ones
- 3 hundreds + 4 tens + 12 ones

Vocabulary: place value, hundreds, tens, ones, compose, decompose

Terminology
- Ten tens equal one hundred.
- One ten is comprised of ten ones; one hundred is comprised of ten tens
- Compose/decompose three-digit numbers using place value of hundreds, tens, ones

Examples of Alternative Algorithms

Educational research has shown that teaching the standard U.S. algorithms are unsuccessful with large numbers of students, and that alternative algorithms are often easier for students to understand and learn. For this reason, The Common Core Standards of Mathematics introduces students to a variety of alternative procedures in addition and subtraction to the customary algorithms. These algorithms and strategies are helpful in building conceptual knowledge students need in order to become proficient mathematical thinkers. Students learn to compose and decompose large numbers with efficiency. Examples of alternative algorithms are Partial Algorithms and Place Value Algorithms.

Examples of Alternative Algorithms

Addition Partial – Sums Algorithm (horizontal)

17 + 25 = ?

(10 + 7) + (20 + 5)

- Add the 10s 10 + 20 = 30
- Add the 1s 7 + 5 = 12
• Add the partial sums \( 30 + 12 = 42 \)

So, \( 17 + 25 = 42 \)

Addition Partial - Sums Algorithm (vertical)
\[
268 + 483 = ? \\
\hline \\
2&6&8 \\
+4&8&3 \\
\hline \\
6&5&1 \\
\hline \\
\]

• Add 100s 6 0 0
• Add 10s 1 4 0
• Add 1s 1 1
• Add partial sums 6 5 1

Subtraction Partial - Sums Algorithm (horizontal)
\[
2457 - 1989 = (2000 - 1000) + (400 - 900) + (50 - 80) + (7 - 9) \\
1000 - 500 - 30 - 2 \\
500 - 30 - 2 \\
So, 470 - 2 = 468 \\
\]

Students who have a strong understanding of subtraction will be able to subtract the larger number from the smaller number regardless of its position in the problem as well as understand whether the answer is positive or negative.

Subtraction Partial - Sums Algorithm (vertical)
\[
835 - 472 = ? \\
\hline \\
8&3&5 \\
-4&7&2 \\
\hline \\
3&6&3 \\
\hline \\
\]

• Subtract the 100s \((800 - 400)\) 400
• Subtract the 10s \((30 - 70)\) - 40
• Subtract the 1s \((5 - 2)\) 3
• Find the total \((400 - 40 + 3)\) 363

Place Value Algorithm Samples

Place Value Column Addition
Column Addition with trading, vertical lines are drawn to divide place values.
\[
796 \\
+451 \\
\hline \\
\]

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<th>Tens</th>
<th>Ones</th>
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<tr>
<td>7</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>+4</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>7</td>
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1,247
First, each column is added individually beginning with the ones.
If necessary, adjustments must be made through trades.
In this case, the hundreds and tens columns need adjusting because we must take a "100" from the tens column; and add it to the hundreds column.
After trading correctly, we find that 796 + 451 equals 1,247.
This method is more significant for students because they are more able to recognize place value in their “trading”.

Place Value Column Subtraction

Column Subtraction with no trading, vertical lines are drawn to divide place values.

\[
\begin{array}{ccc}
 7 & 9 & 6 \\
- & 4 & 5 & 1 \\
\hline
 3 & 4 & 5
\end{array}
\]

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</tr>
<tr>
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345

• First, each column is subtracted individually beginning with the ones.
• If necessary, adjustments must be made through trades.
• In this case, no trading adjustments need to be made.

Column Subtraction with trading, vertical lines are drawn to divide place values.

\[
\begin{array}{ccc}
 7 & 5 & 6 \\
- & 4 & 9 & 1 \\
\hline
 2 & 6 & 5
\end{array}
\]

\[
\begin{array}{ccc}
 7 & 6 & 6 \\
- & 4 & 9 & 1 \\
\hline
 2 & 6 & 5
\end{array}
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265

• First, each column is subtracted individually beginning with the ones.
• If necessary, adjustments must be made through trades.
• In this case, the tens column need adjusting because we must take a "100" from the hundreds column; and add it to tens column.
• After trading correctly, we find that 756 - 491 equals 265.
• This method is more significant for students because they are more able to recognize place value in their “trading”.